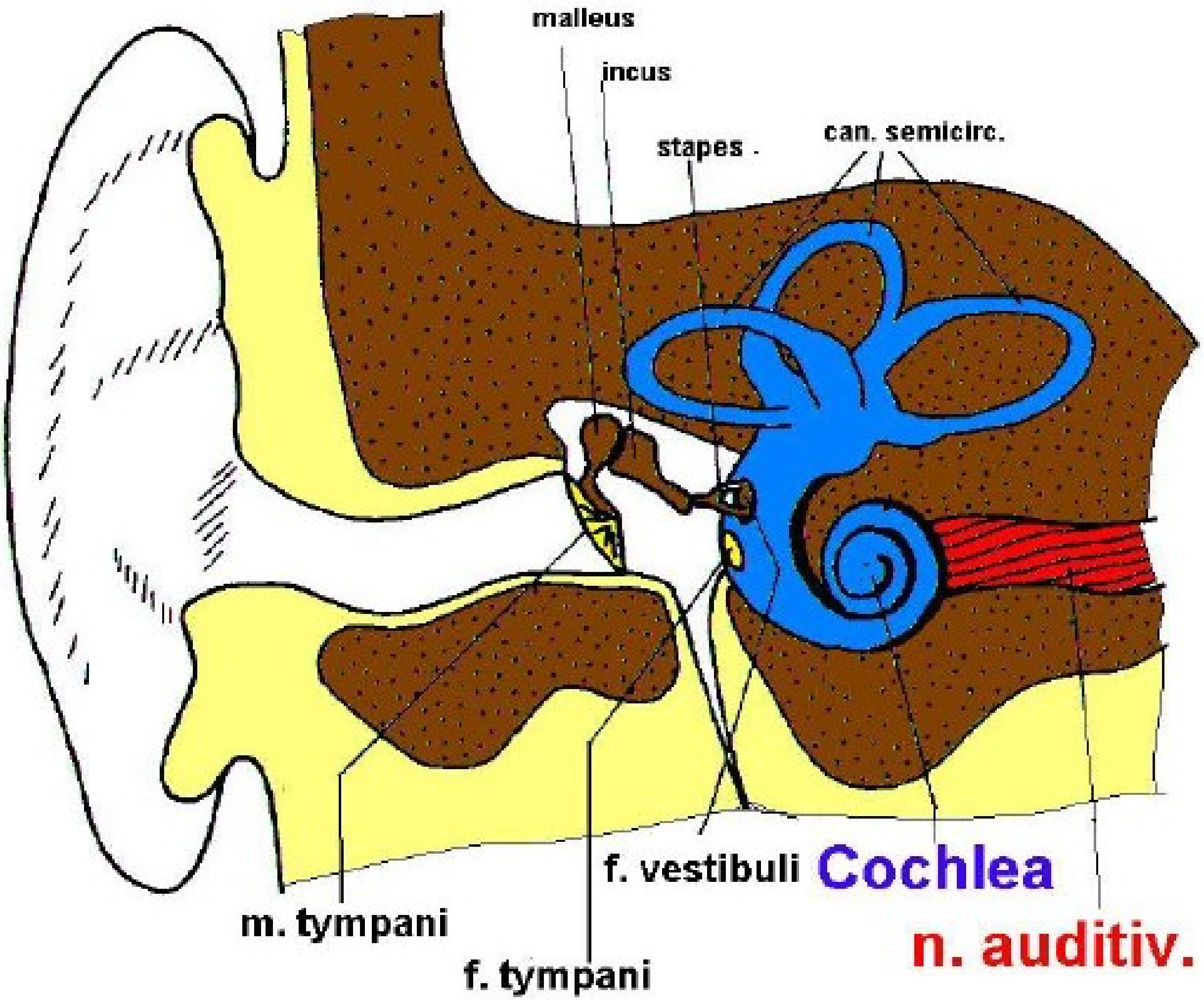
II The Physics of the Ear

1) Anatomy of the Ear

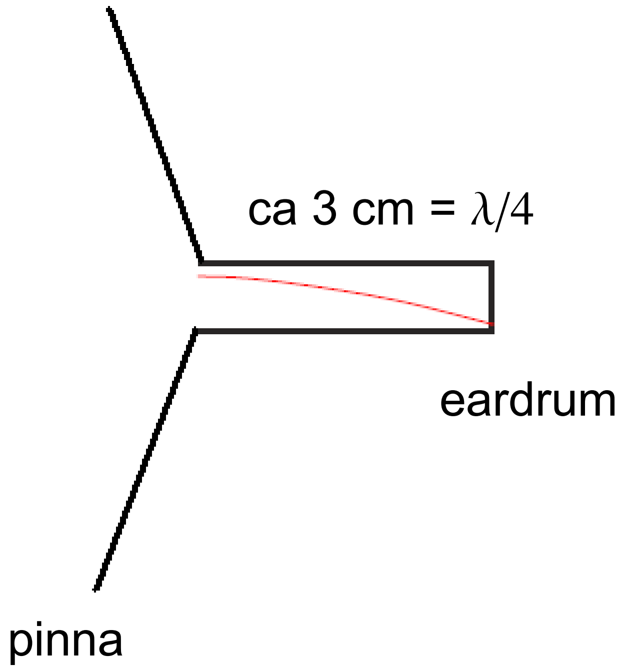
2) Hydrodynamics of the Cochlea

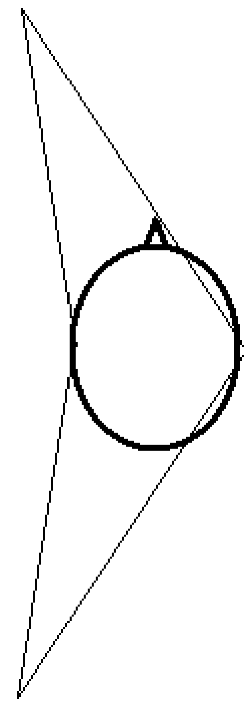
3) Realtion to Psychophysics

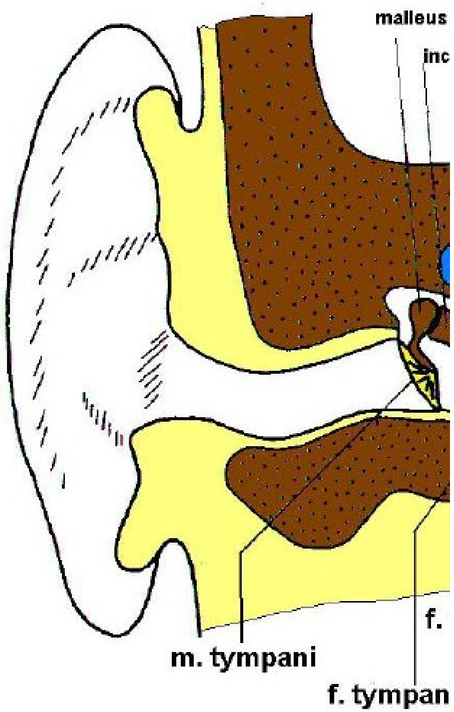
The human ear



The external ear







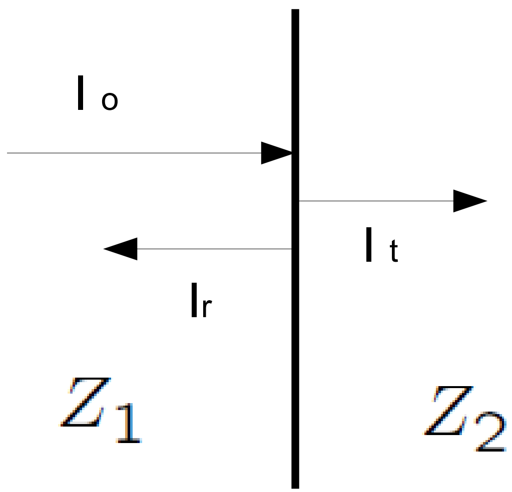
Collects sound (funnel-effect)
dicriminates between front and back
Enforces sound at $\lambda = 4 * 3$ cm , ca 3000 Hz

The middle ear

Impedance matching.

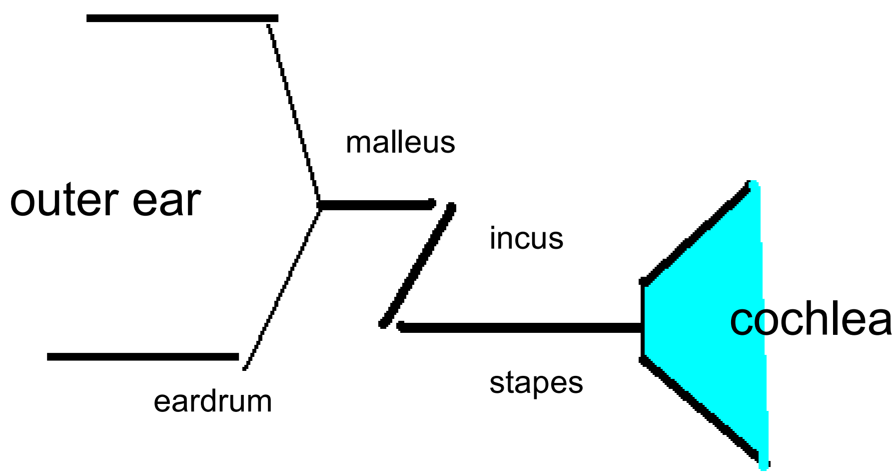
In air: large displacement of the matter (molecules) low impedance
$Z = c\rho = 450\ kg/(m^2\ s)$

In liquid : small displacement, high impedance, Z= 1 500 000 *kg/(m s)*

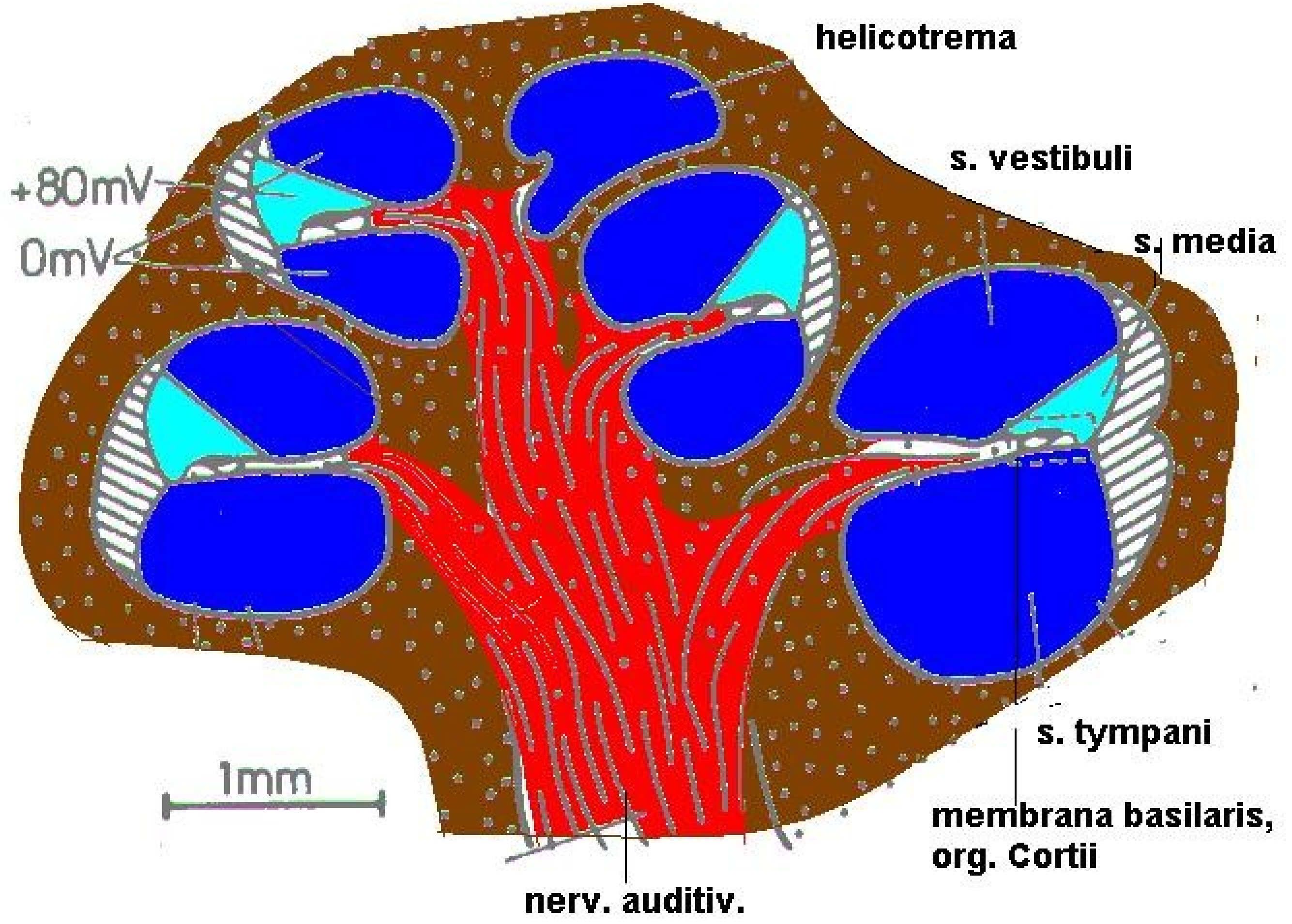


for stationary waves:

$$\frac{I_t}{I_0} = 1 - \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

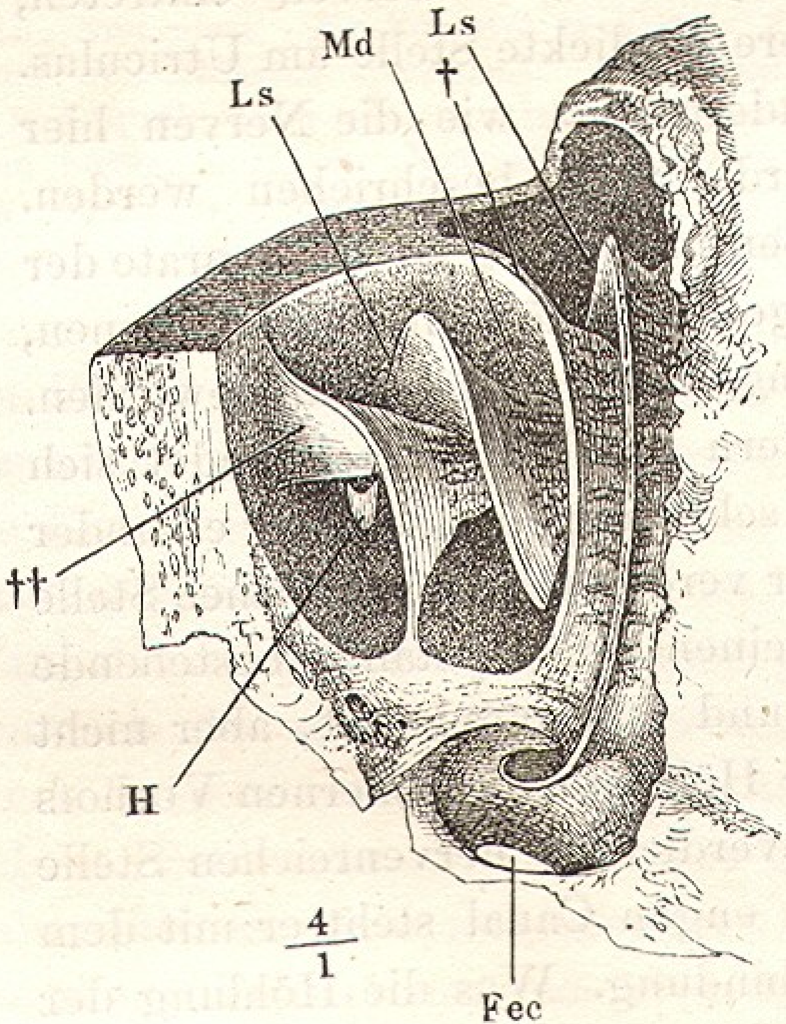


Impedance mismatch for traveling wave ca 360, reduced by factor 90 by the middle ear

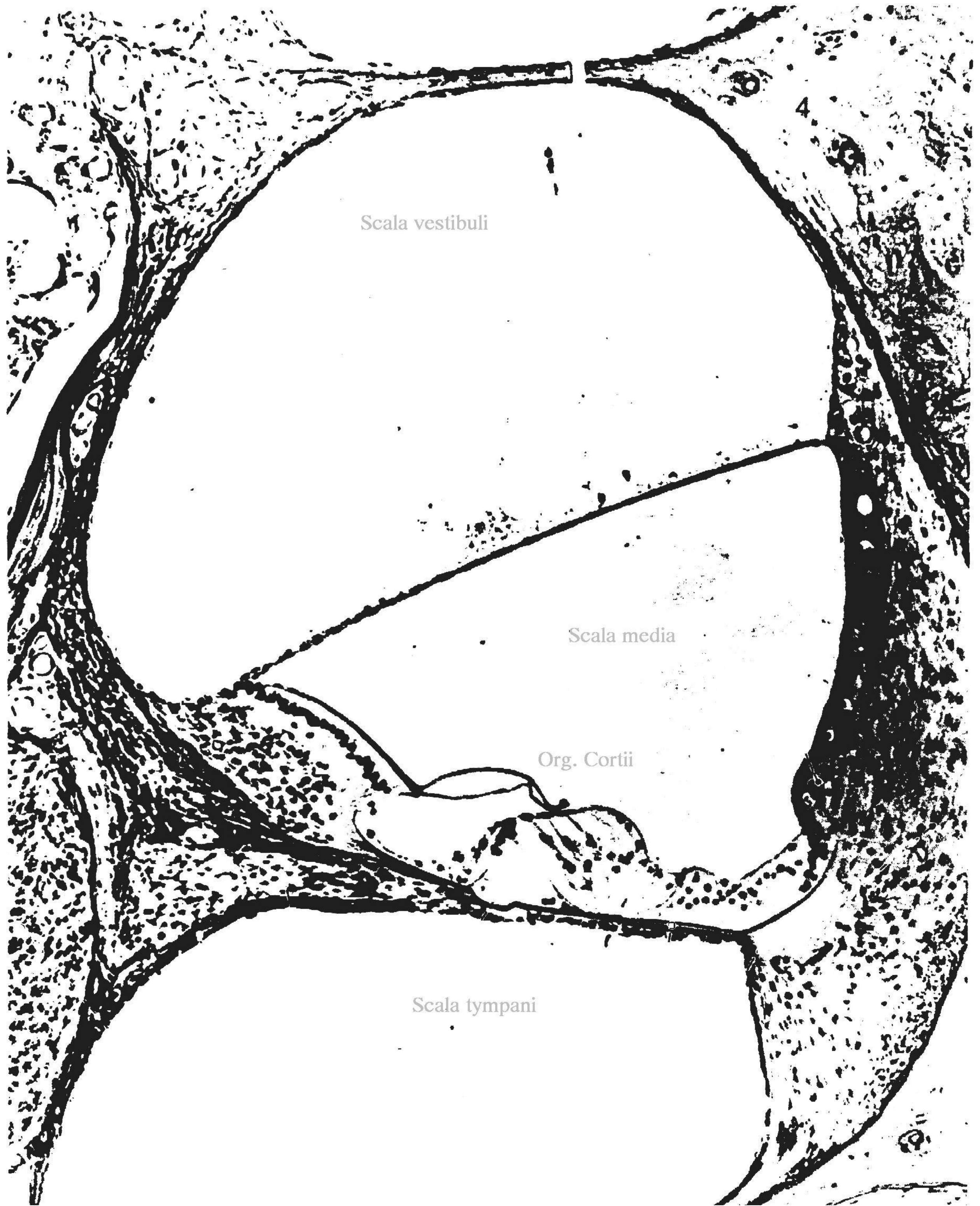

helicotrema
s. vestibuli
s. media
+80mV
0mV
s. tympani
membrana basilaris,
org. Cortii
1mm
nerv. auditiv.

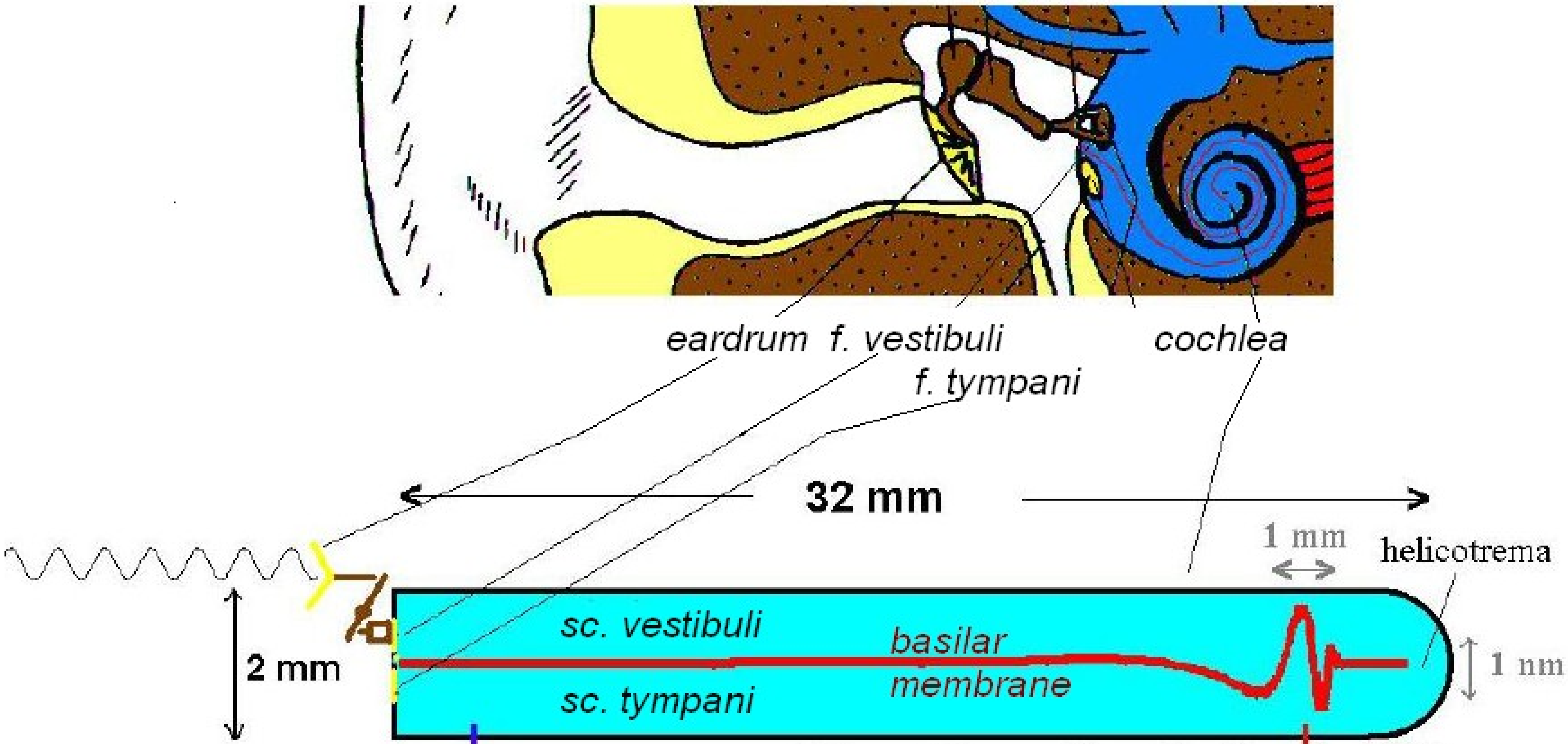
Drawing of the cochlea from Helmholtz
Lehre von den Tonempfindungen

Fig. 45.

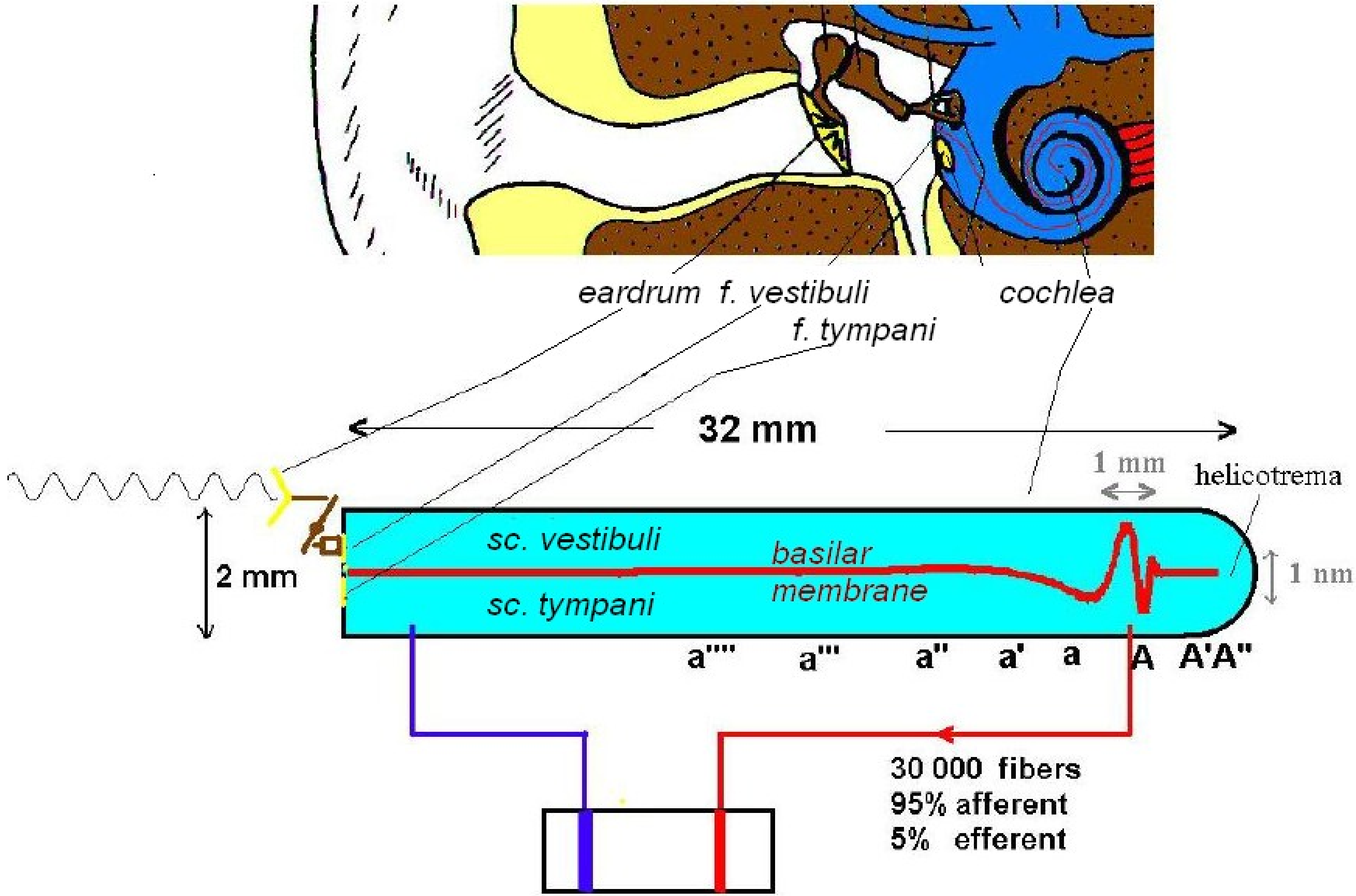


Knöcherne (rechte) Schnecke, von vorn geöffnet. *Md* Modiolus. *Ls* Lamina spiralis. *H* Hamulus. *Fec* Fenestra cochleae. † Durchschnitt der Zwischenwand der Schnecke. †† Oberes Ende derselben.


Scala vestibuli
4
Scala media
Org. Cortii
Scala tympani



Conjecture of Helmholtz:
Different places of the bm are sensitive to different frequencies (filter bank).


eardrum
f. vestibuli
f. tympani
cochlea
32 mm
1 mm
helicotrema
sc. vestibuli
basilar
membrane
sc. tympani
2 mm
1 nm
a'''' a''' a'' a' a A A'A''
30 000 fibers
95% afferent
5% efferent

V. Bekesy

Experiments in Hearing

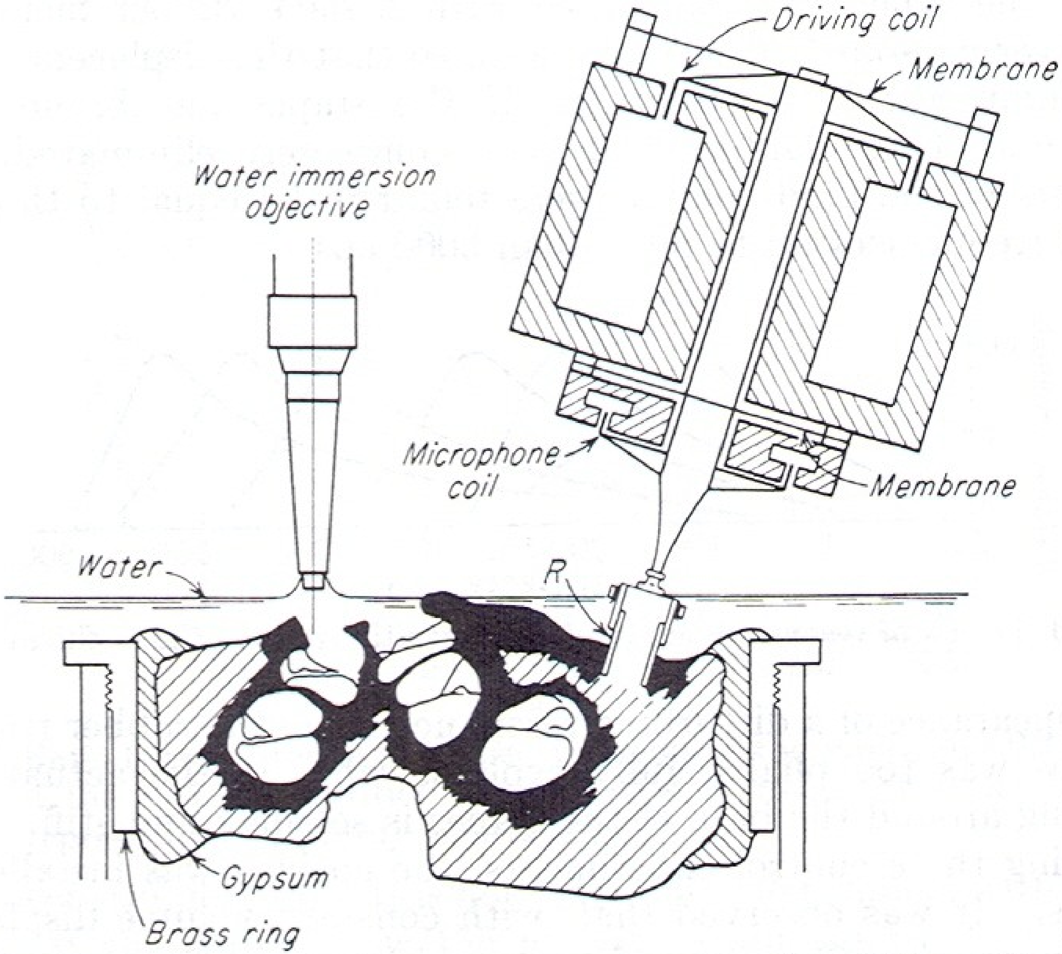


Fig. 11-48. Method of measuring the amplitude of vibration of the cochlear partition in response to volume displacements of the stapes.

[Bek60] Bekesy, Georg von. *Experiments in Hearing*. McGraw-Hill, 1960.

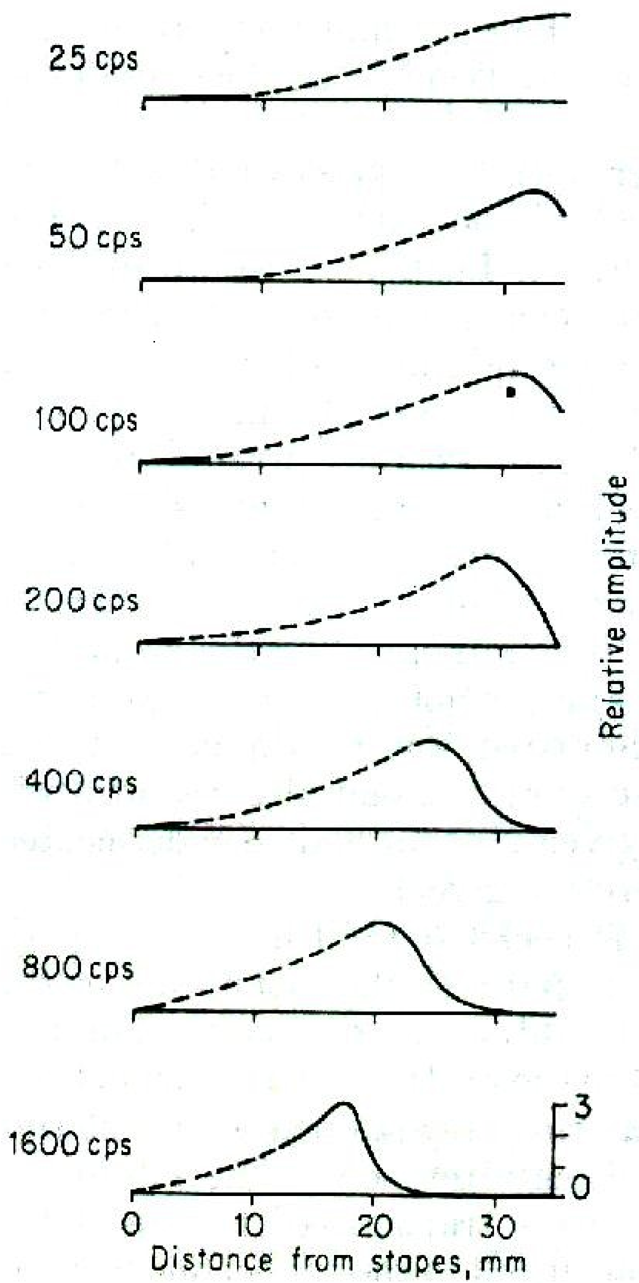


FIG. 11-43. Patterns of vibration of the cochlear partition of a cadaver specimen for various frequencies.

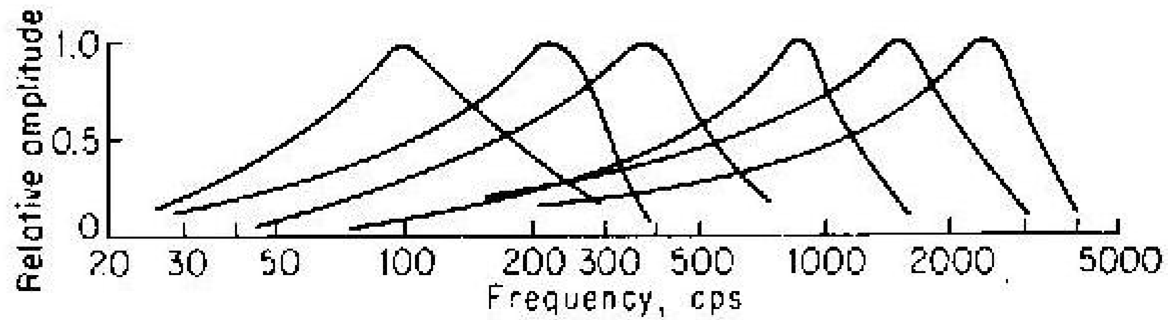
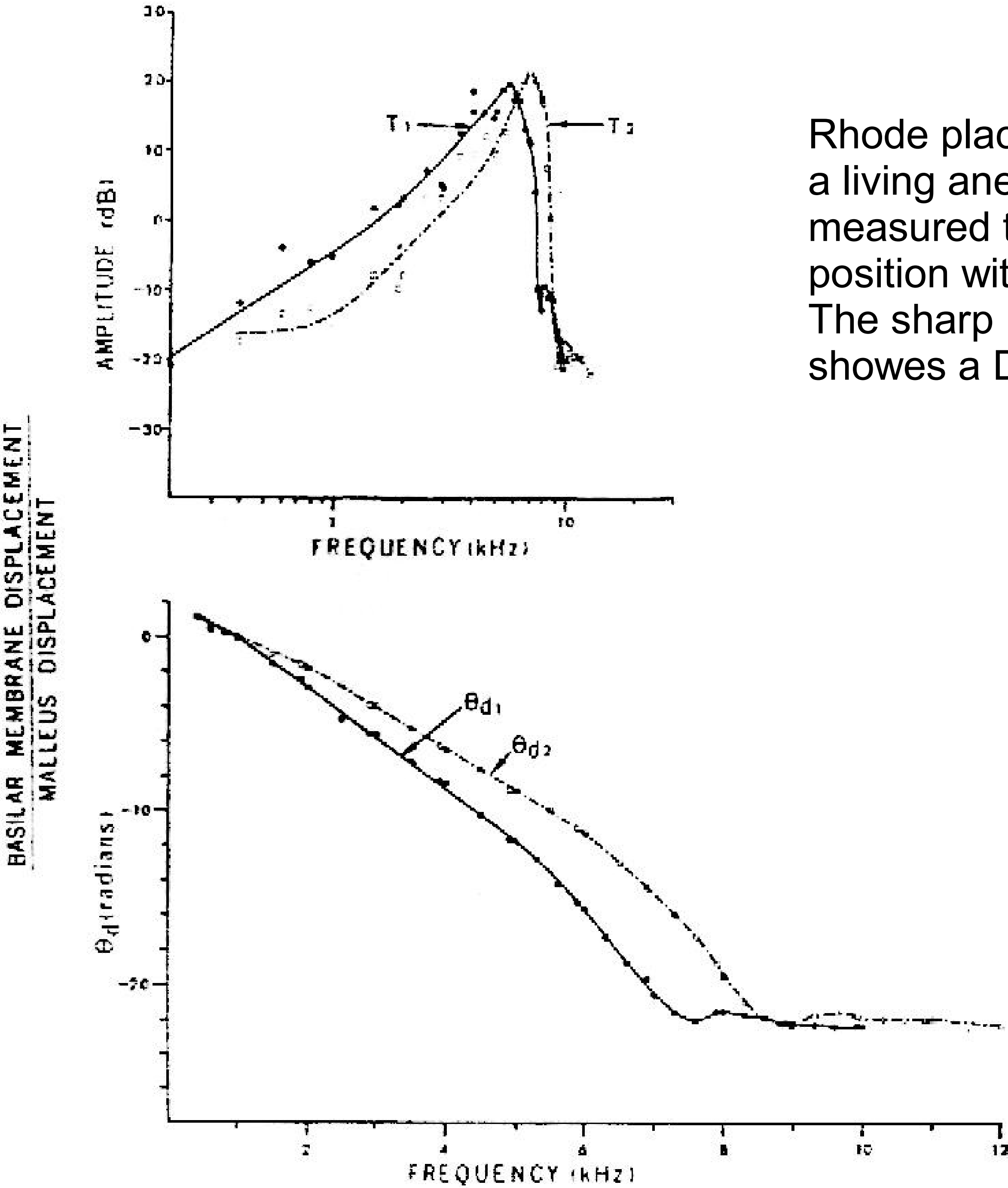
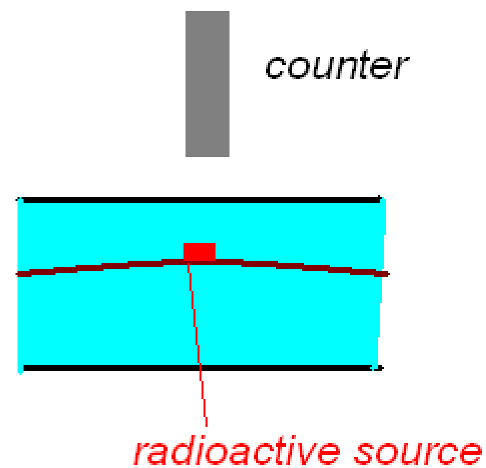


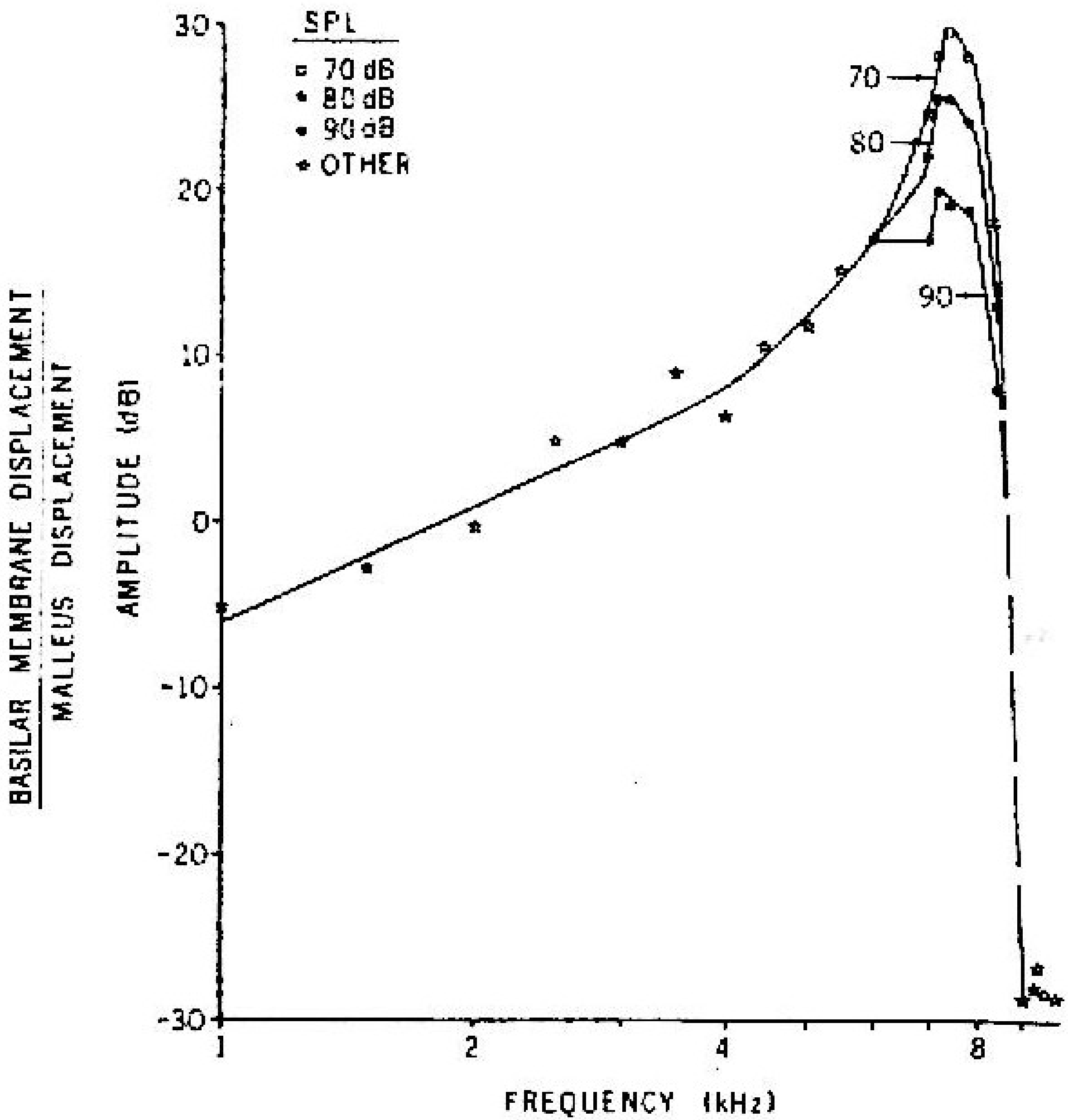
Fig. 11-49 Forms of resonance curves for six positions along the cochlear partition



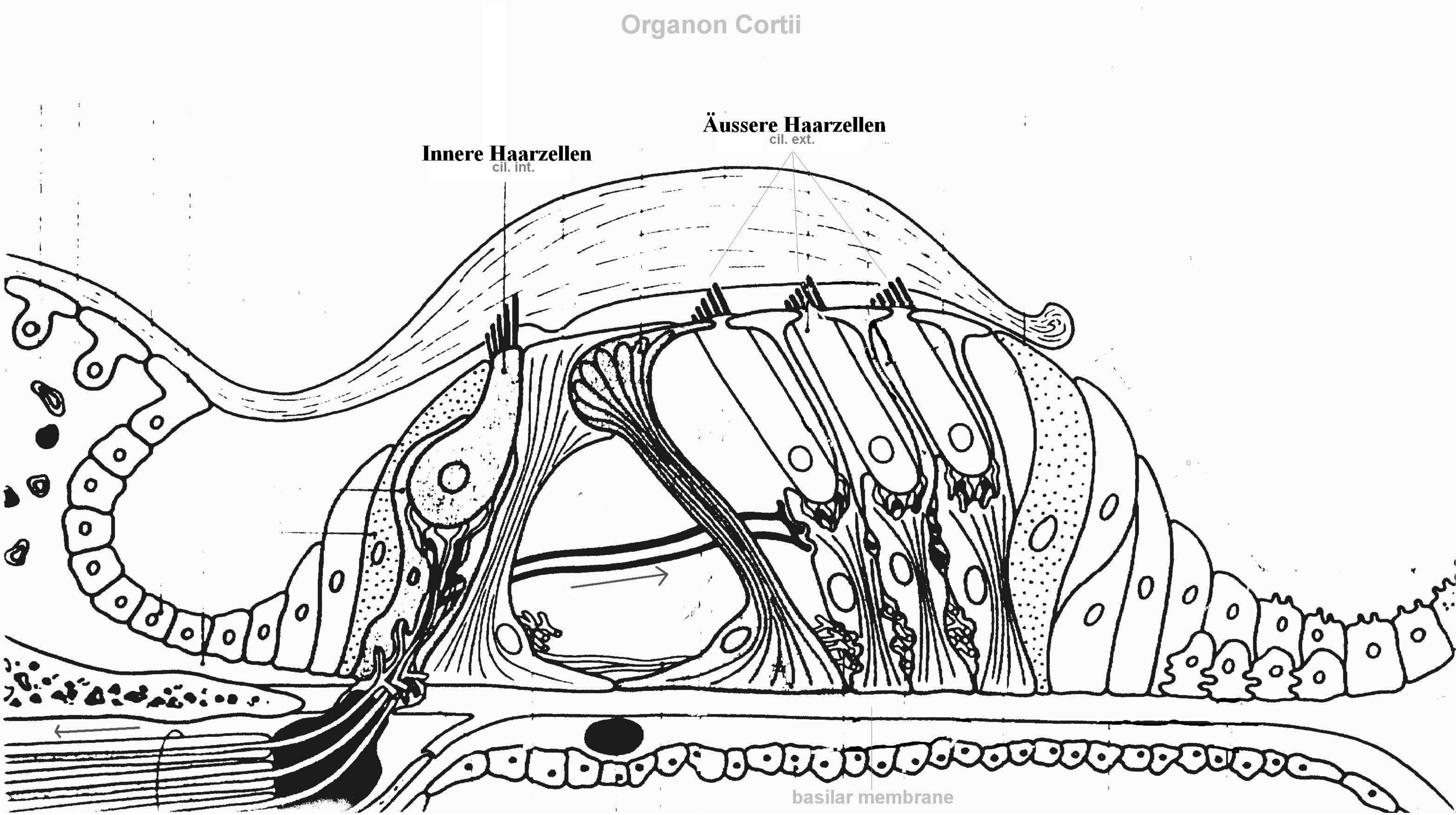
Rhode placed a radioctive source on the bm of a living anethesized squirrel monkey and measured the vibration of the bm at this position with the help of the Moessbauer effect: The sharp line of the radioactive source showes a Doppler effect due to the oscillation.



[Rho71] W.S. Rhode. Observation of the vibration of the basilar membrane in squirrel monkeys using the mössbauer technique. *JASA*, 49:1218, 1971.

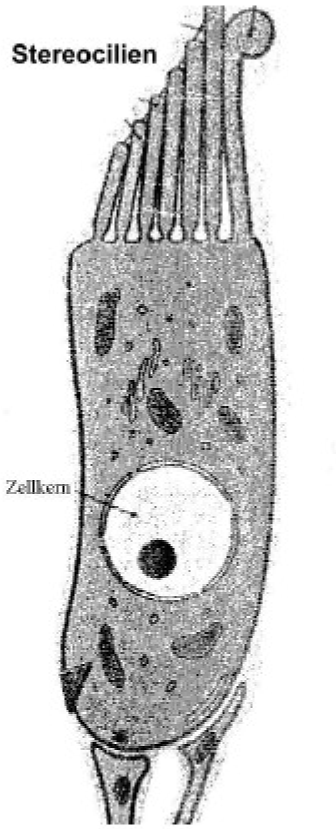


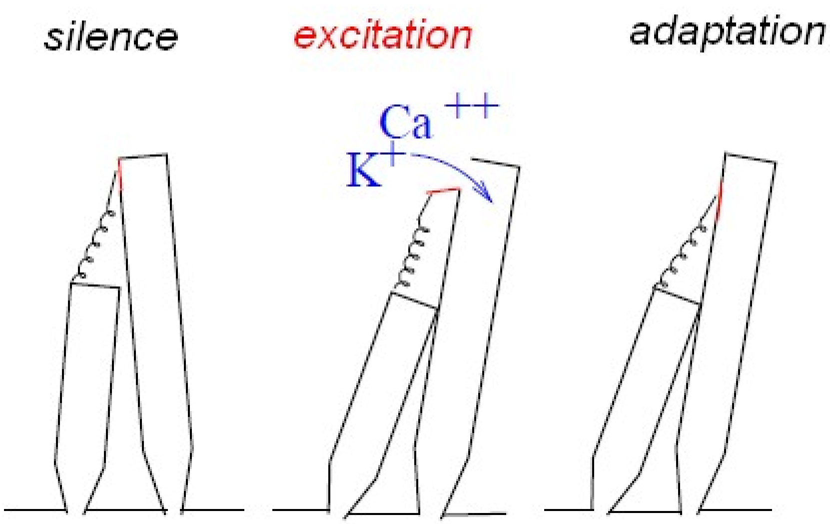
The reaction of the bm to the SPL is highly non-linear. The ratio of the displacement of the bm to that of the malleus (eardrum) decreases strongly with the SPL

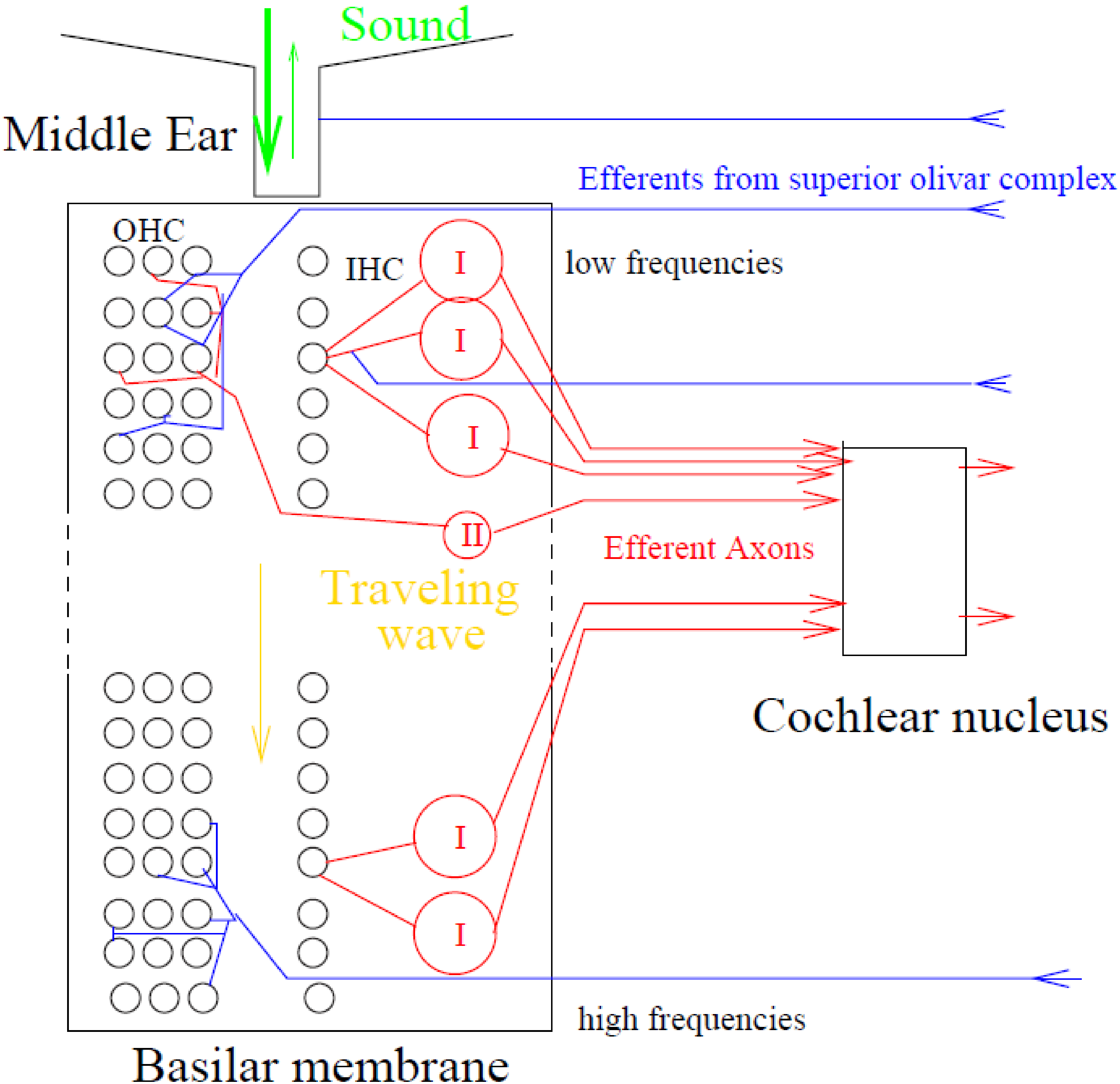


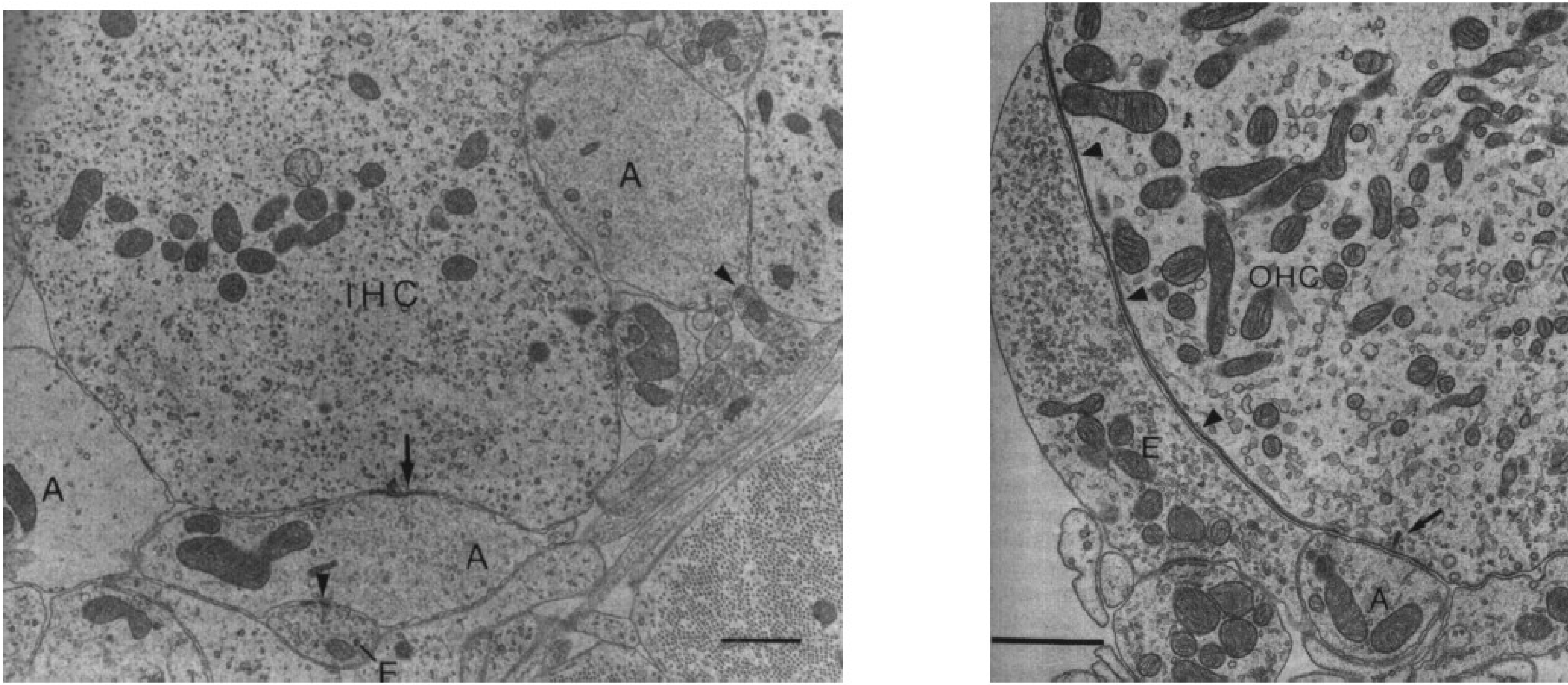
Organon Cortii
Äussere Haarzellen
cil. ext.
Innere Haarzellen
cil. int.
basilar membrane

Probable mechanism of signal generation by inner hair cells






Sound
Middle Ear
Efferents from superior olivar complex
OHC
IHC
I
I
I
II
low frequencies
Efferent Axons
Traveling
wave
Cochlear nucleus
I
I
high frequencies
Basilar membrane



The connection of efferent (E) and afferent (A) nerves with a inner hair cell (IHC, left) and an outer hair cell (OHC, right). the synapses are marked by arrows

[Ryu92] D.K. Ryugo. The auditory nerve. In D.B. Webster, A.N. Popper, and R.R. Fay, editors, *The Mammalian Auditory Pathway: Neuroanatomy*. Springer, Berlin Heidelberg New York, 1992.

Hydrodynamics of the cochlea

This is the mathematically most demanding part of the lectures,
see also pdf-file cochlea-dynamics on the home page.

1) The mathematical problem

2) Some intuitive considerations

3) Solution of a special model

Newton: $\rho\frac{d\vec{v}}{dt} = -\vec{\partial}p$, or $\rho\frac{dv_x}{dt} = -\vec{\partial}_x p$, $\rho\frac{dv_x}{dt} = -\vec{\partial}_y p$

Conservation of matter: $\partial_t\rho = -\vec{\partial}\cdot(\rho\,\vec{v}) \equiv \partial_x\rho\,v_x + \partial_y\,\rho v_y$

Simplifications:

1) $\frac{dv}{dt} \approx \partial_t v$ (velocity small)

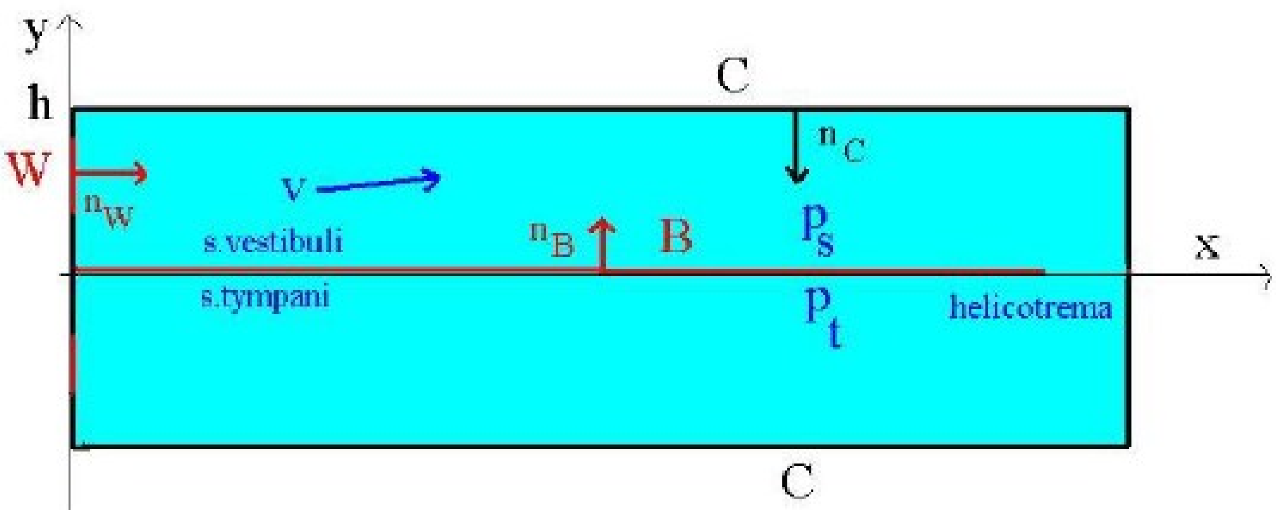
2) $\rho = \text{const} \;\rightarrow\; \vec{\partial}\cdot\vec{v} \equiv \partial_x v_x + \partial_y v_y = 0$

matter incompressible, justified if $\lambda = c/\nu \gg 0.03m \rightarrow \nu \ll 50\,000$ Hz

Then follows

$$\partial^2 p = -\rho\partial_t\vec{\partial}\cdot\vec{v} = 0$$

Difficult problem: solve the boundary conditions:



$$\begin{aligned}
\vec{n}_W.\vec{v} &= w \text{ on } W \\
\vec{n}_C.\vec{v} &= 0 \text{ on } C \\
\vec{n}_B.\vec{v} &= \partial_t Y \text{ on } B \\
p &= 0 \text{ on the helicotrema}
\end{aligned}$$

$$\rho\dot{\vec{v}} = -\vec{\partial}p$$

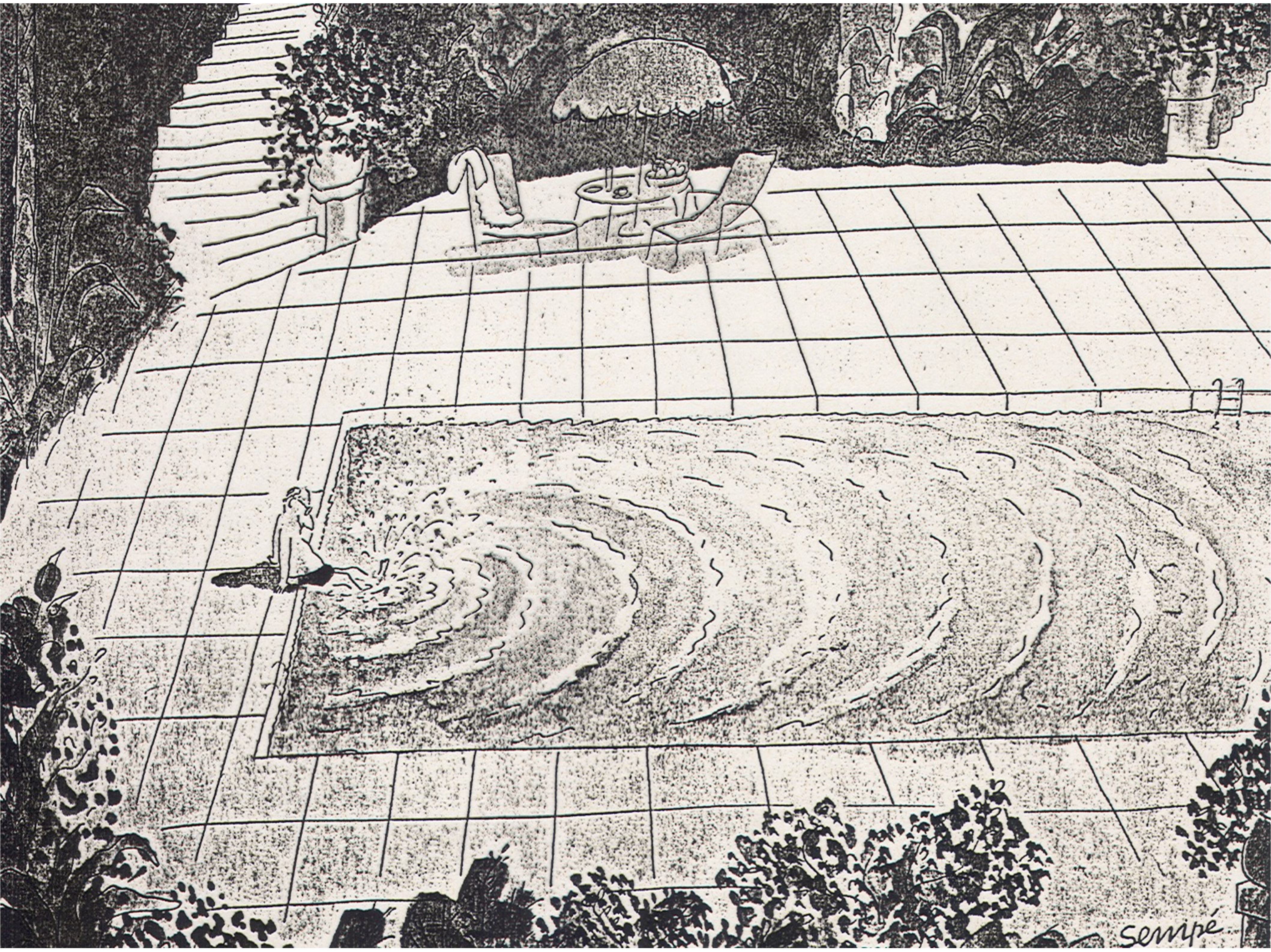
properties of the bm:

$$p = F(Y, \partial_t Y, \partial_t^2 Y, x) \text{ on } B$$

Qualitative considerations:

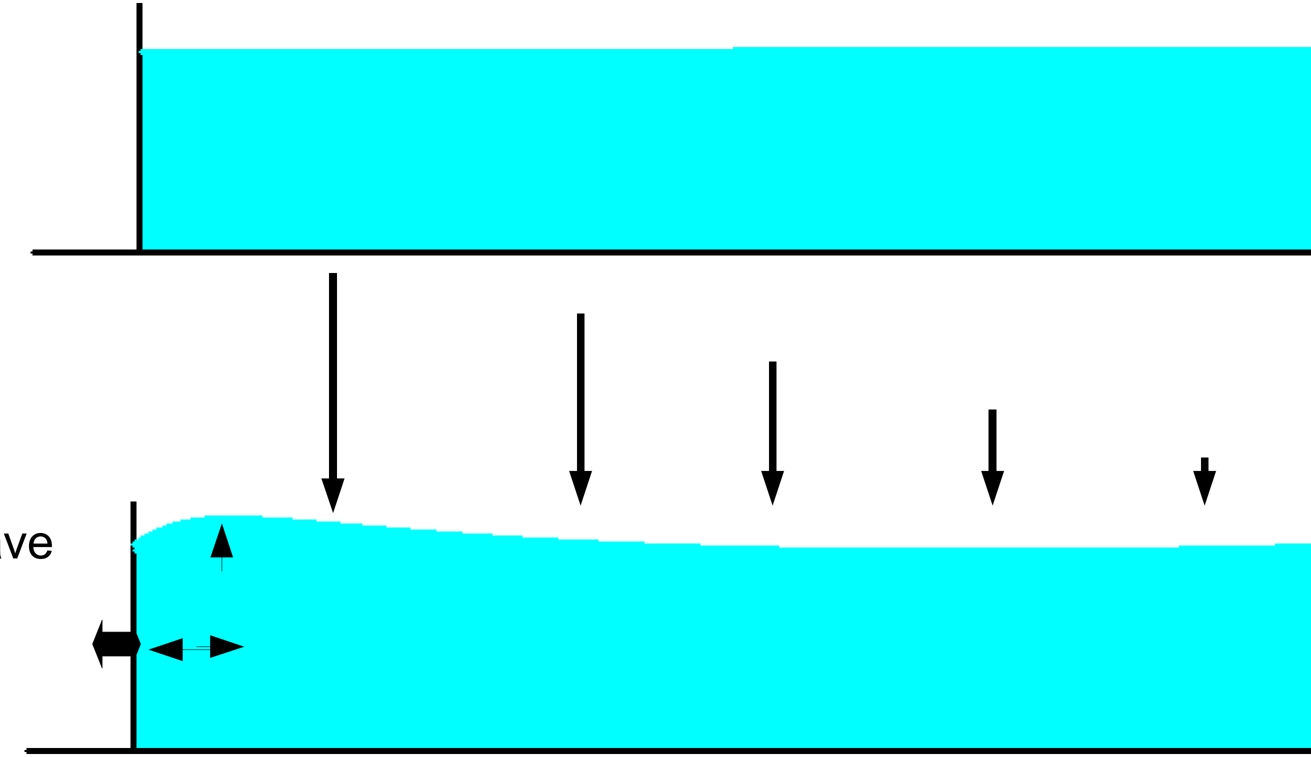
traveling shallow water wave

in water waves : gravity pushing the water down

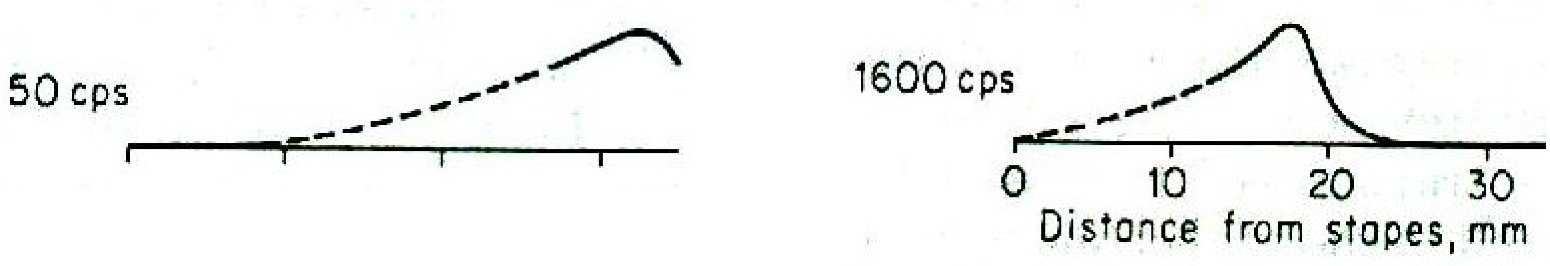

Sempé

Qualitative considerations:

traveling shallow water wave



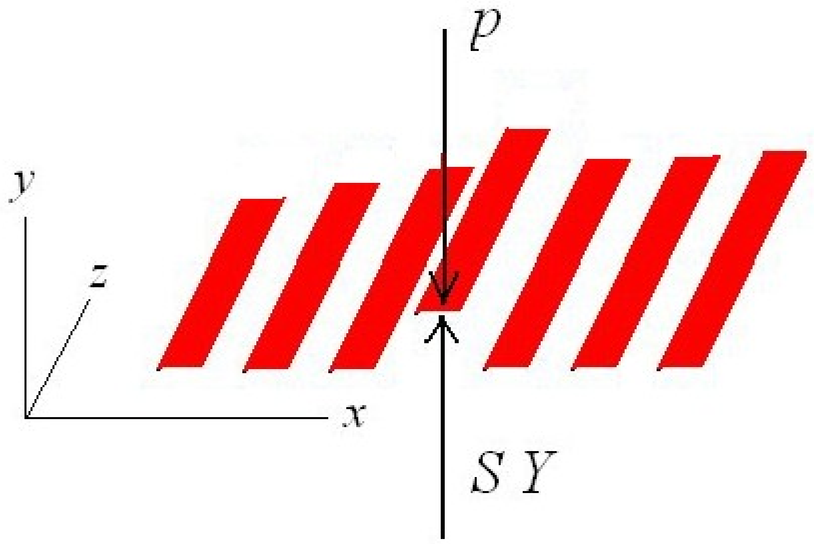
in water waves : gravity, in cochlea: bm pushing back
When frequency matches the resonance frequency of a part of the bm, then there largest excitation and wave looses its energy.



Anatomy of the bm:

narrow and stiff near stapes (f. vestibuli), and slack and broad near helicotrema.

A not very realistic, but widely used and instructive model for the bm: p is linear functional of the pressure, e.g.



$$m\partial_t^2 Y = -SY - R\partial_t Y - p$$

i.e. locally the bm acts as a damped harmonic oscillator driven by the pressure

$$p(x,y) = \int \frac{dk}{2\pi} \tilde{p}(k,y) e^{-ikx} \quad \longrightarrow \quad \partial_y^2 \tilde{p}(k,y) = -k^2 \tilde{p}(k,y)$$

$$\vec{\partial}^{\,2} p = -\rho \partial_t \vec{\partial}.\vec{v} = 0$$

$$\tilde{p}(k,y) = A(k)\, e^{k\,y} + B(k)\, e^{-k\,y} \quad \longrightarrow \quad \tilde{p}(k,y) = \tilde{p}(k) \frac{\cosh[k(h-y)]}{\cosh[kh]}$$

$v_y = 0$ at $y = h$ follows $\partial_y p(k,h) = 0$

$$v_y(x,0,t) = \partial_t Y(x,t) \qquad \vec{\partial} p = -\rho \partial_t \vec{v}$$

$$\longrightarrow \quad -\rho \partial_t^2 Y(x,t) = \int_0^\infty \frac{dk}{2\pi} \tilde{p}(k) k \tanh[k\,h] e^{ik\,x}$$

$$\swarrow$$

$$-\rho \partial_t Y(x,t) = D_Q\, p[Y,x,t]$$

$$D_Q = \left(\begin{array}{lll} D_Q^L \equiv -h\partial_x^2 & \text{for } \; k\,h & < 1 \\ D_Q^S \equiv -i\partial_x & \text{for } \; k\,h & > 1 \end{array} \right.$$

$$kh \ll 1 \;: \quad -\rho \partial_t Y(x,t) = -h\, \partial_x^2 p(x,Y,t)$$

$$m\partial_t^2 Y = -SY - R\partial_t Y - p \longrightarrow -m\omega^2 Y(x) = -S(x)Y + i\omega R(x)Y(x) - 2p(x,0)$$

$$Y(x,t) = Y(x)e^{-i\omega t}$$

$$-\rho\partial_t Y(x,t) = -h\,\partial_x^2 p(x,Y,t) \longrightarrow i\omega\rho Y(x) = -h\partial_x^2 p(x,Y)$$

$$\partial_x^2 p(x,0) = \frac{-2\rho\omega^2}{S(x) - m\omega^2 - i\omega R(x)} p(x,0)$$

can be solved numerically, good approximation: WKB approximation:

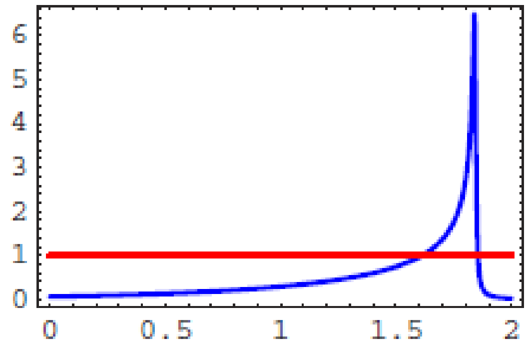
p(x,Y)= $A\frac{1}{\sqrt{g(x)}}\exp[i\int_0^x dx'\, g(x')]$ with $g(x) = \omega\sqrt{\frac{2\rho}{S(x) - m\omega^2 - i\omega R(x)}}$

This is exact for *g(x)* = const, good approximation for slowly varying g.

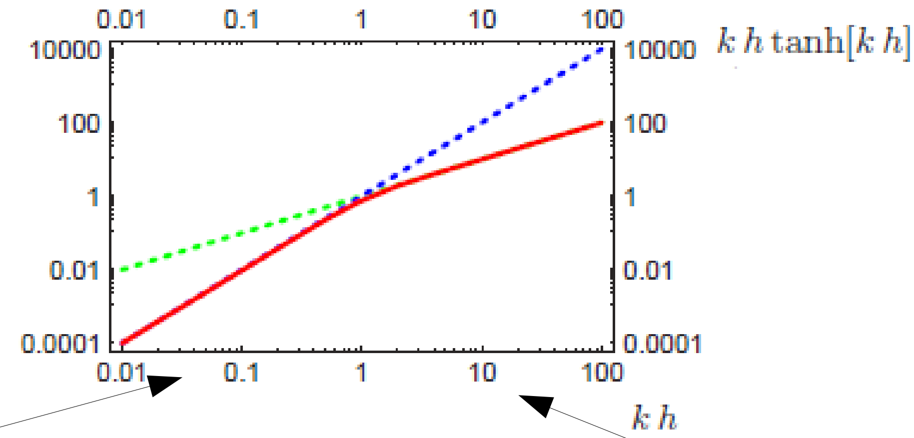
$$p(x,0) = A\frac{1}{\sqrt{g(x)}}\exp[i\int_0^x dx'\, g(x')] \approx A' e^{ig(x_0)(x-x_0)} \text{ for } x \approx x_0$$

local wave vector: $k(x) = g(x) = \omega\sqrt{\frac{2\rho}{S(x) - m\omega^2 - i\omega R(x)}}$

Consistency: $kh \ll 1$



The expression $k(x)\,h$ as function of x for $\nu = 1000$ Hz.



way out: for *kh <1* long wave length approximation, (as previous) for *kh>1* short w.l.

$$kh \gg 1 \;:\quad -\rho_b \partial_t Y(x,t) = -h\,\partial_x p(x,Y,t)$$ easy to solve

ρ_b mass density of bm

Parameters of the model:

tension, damping

$$S(x) = C_0 e^{-\alpha x} - a, \quad C_0 = 10^9, \quad \alpha = 3, \quad m = 0.05, \; R = R_0 e^{-\alpha x/2}$$

$$\delta = \frac{R_0}{\sqrt{C_0\, m}}; \quad h = 0.1, \quad \rho = 1 \quad \text{all in CGS units}$$

with δ varying from 0.2 to 0.005.

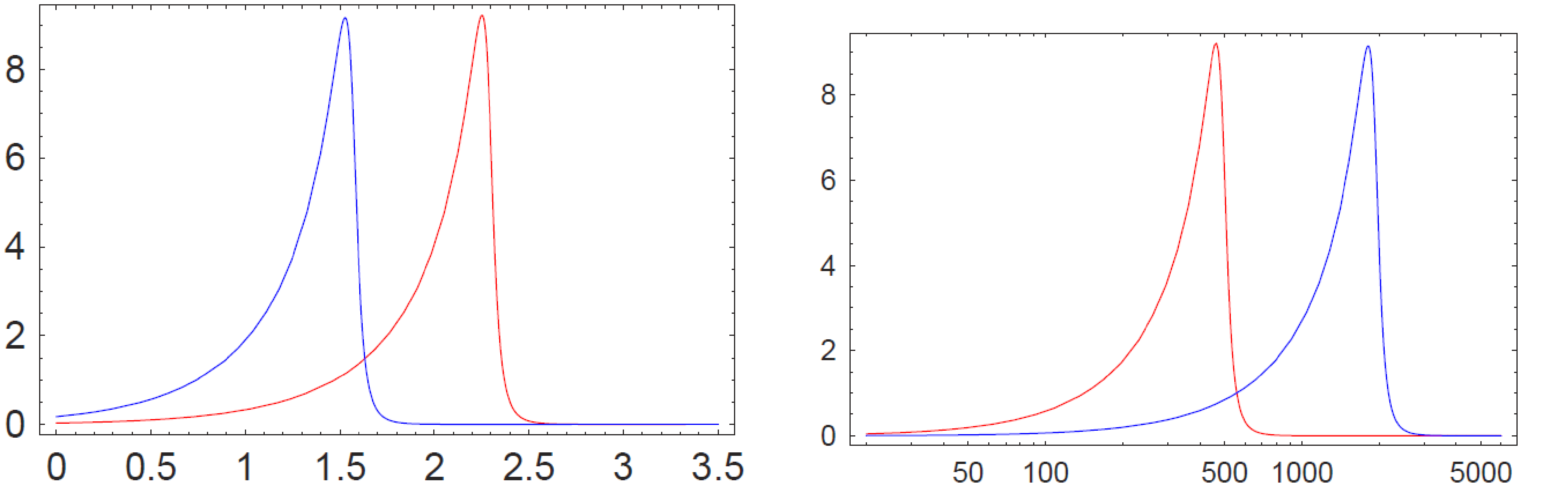
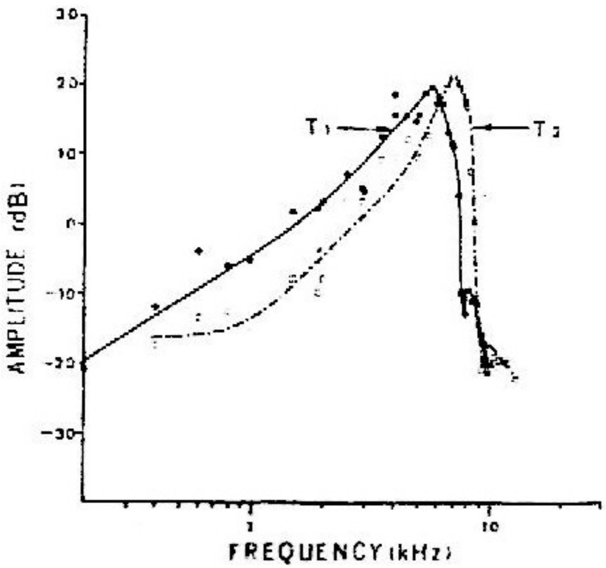
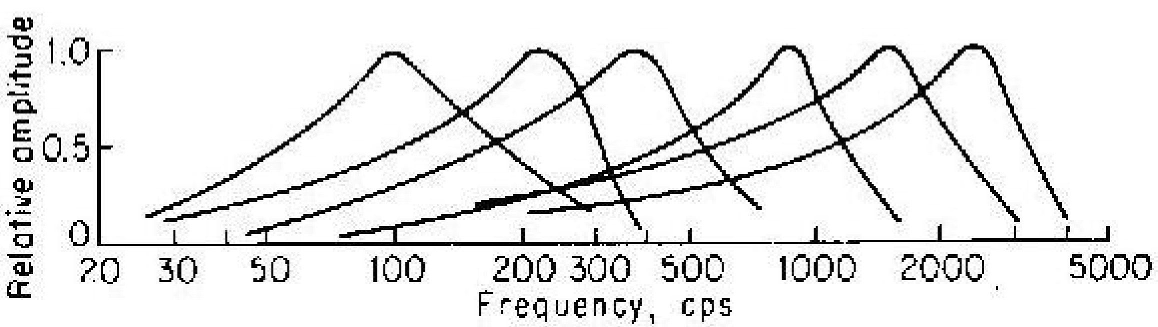


Abbildung 3.16: **Left:** Envelope of the velocity of the BM for two tones of 500 (red) and 1500 Hz (blue) as a function of x **Right :** Envelope of the velocity of the BM for positions of 2.3 (red) and 1.4 cm (blue) as a function of ν





$p_s(t) \;=\; e^{-i\omega t}$ at stapes $\longrightarrow$ $p(x,Y,\omega)$

$\tilde{p}_s(\omega)e^{-i\omega t}$ $\longrightarrow$ $\tilde{p}_s(\omega)\,p(x,Y,\omega)$

general signal $p_s(t)$ $\qquad \tilde{p}_s(\omega) = \int dt\, p_s(t)e^{i\omega t}$

$p_s(t)$ $\longrightarrow$ $\widehat{p}_s(x,Y,t)$ on the bm

$$\widehat{p}_s(x,Y,t) = \int \frac{d\omega}{2\pi}\, p(x,Y,\omega)\,\tilde{p}_s(\omega)e^{-i\omega t}$$

$$= \int \frac{d\omega}{2\pi} \int dt'\, p_s(t')e^{i\omega t'} \int dt''\, \widehat{p}(x,Y,t'')e^{i\omega t''}\, e^{-i\omega t}$$

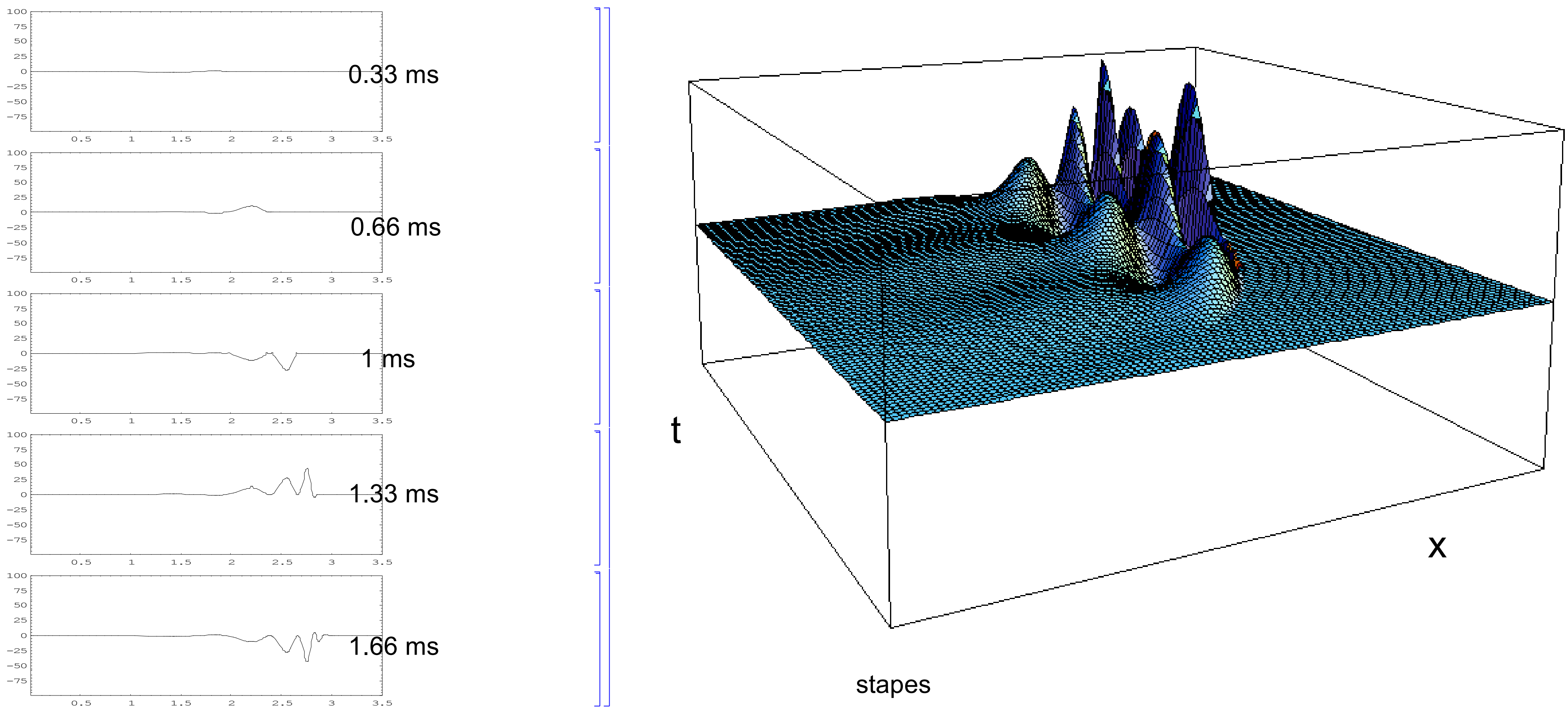
$$= \int dt'\, p_s(t') \int dt''\, \widehat{p}(x,Y,t'')\delta(t''+t'-t)$$

$$= \int dt'\, p_s(t')\widehat{p}(x,Y,t-t')$$

with $\widehat{p}(x,Y,t) = \int \frac{d\omega}{2\pi}\, p(x,Y,\omega)\, e^{-i\omega t}$

Excitation by steady tone $w(t) = \sin(2\pi\nu_0 t)$ leads to movement of the

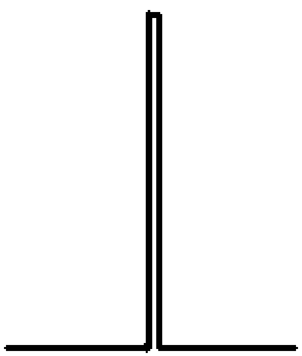
whole bm with this frequency, but with maximal amplitude at the resonance position (chracteristic frequency, CF).



Movement (velocity) of the bm for a tone of 300 Hz

Excitation by
a click at time
0

$$w(t) = \delta(t)$$



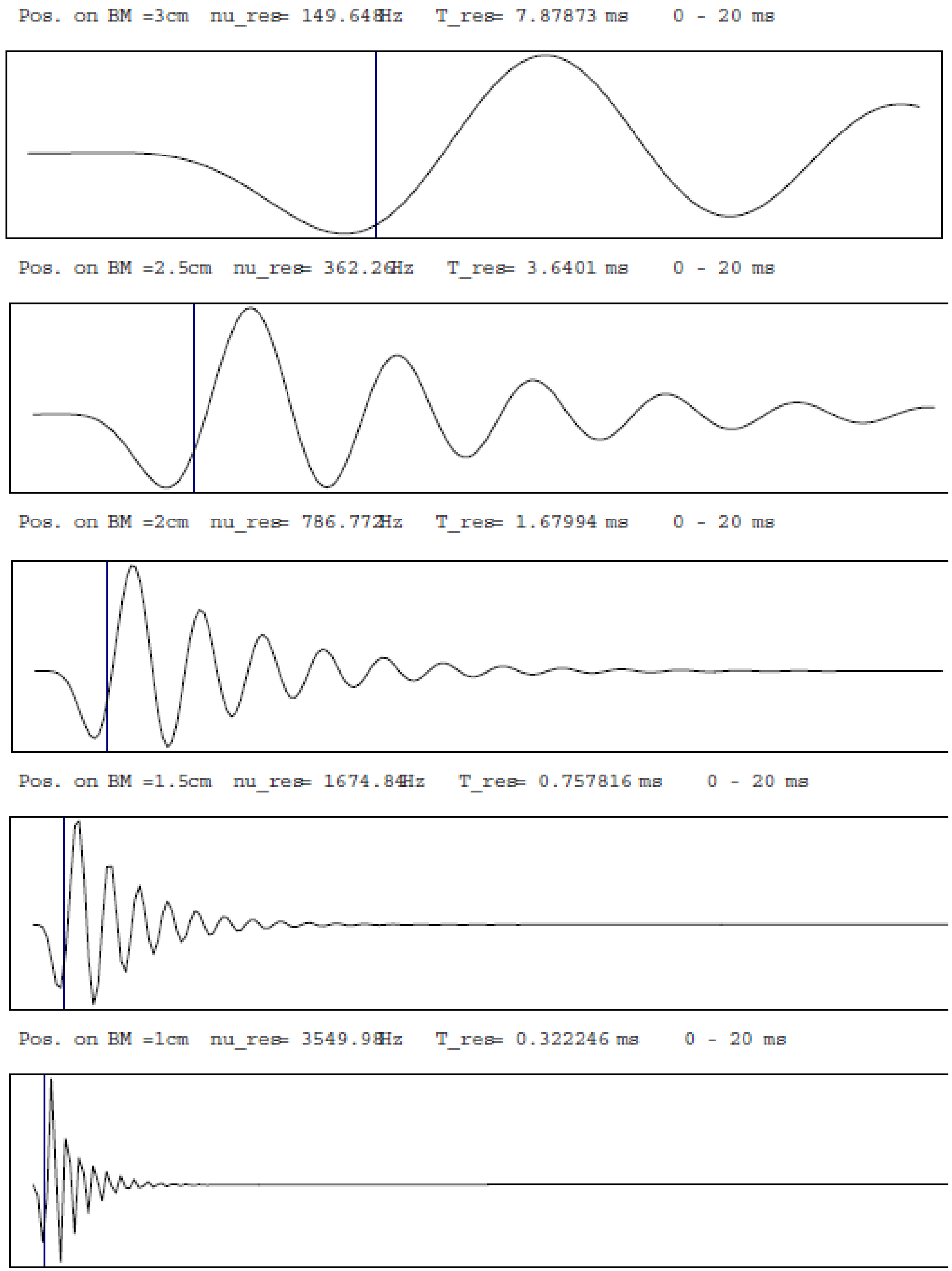
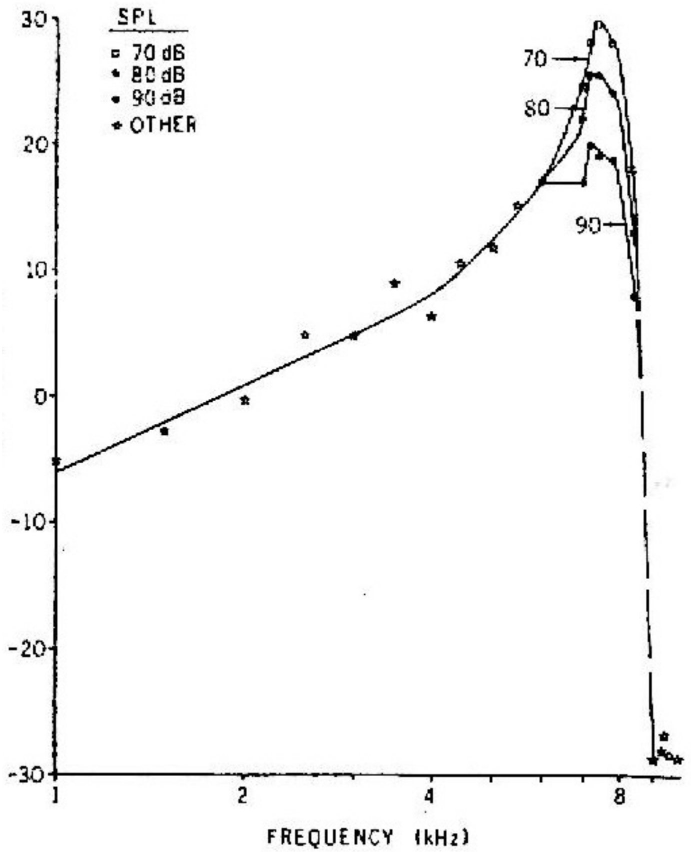


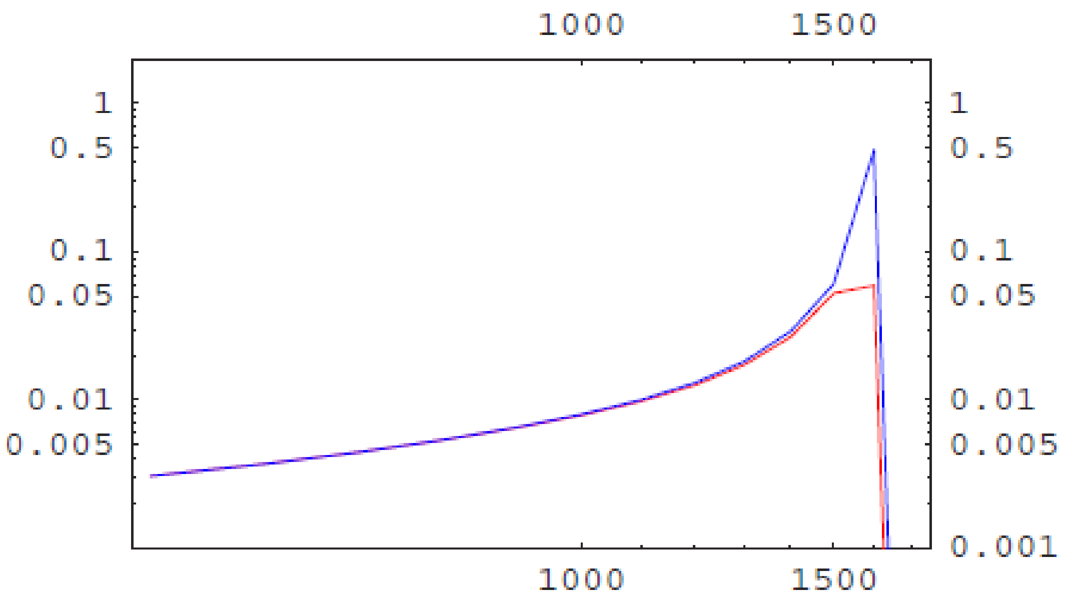
Abbildung 3.17: The amplitude Y of the BM as a function of time (0 to 20 ms) at positions $x =$ 2, 1.5 and 1 cm as a response to a click. The vertical grid line shows the arrival time of the sign calculated from the group velocity.

A linear model is not realistic, see



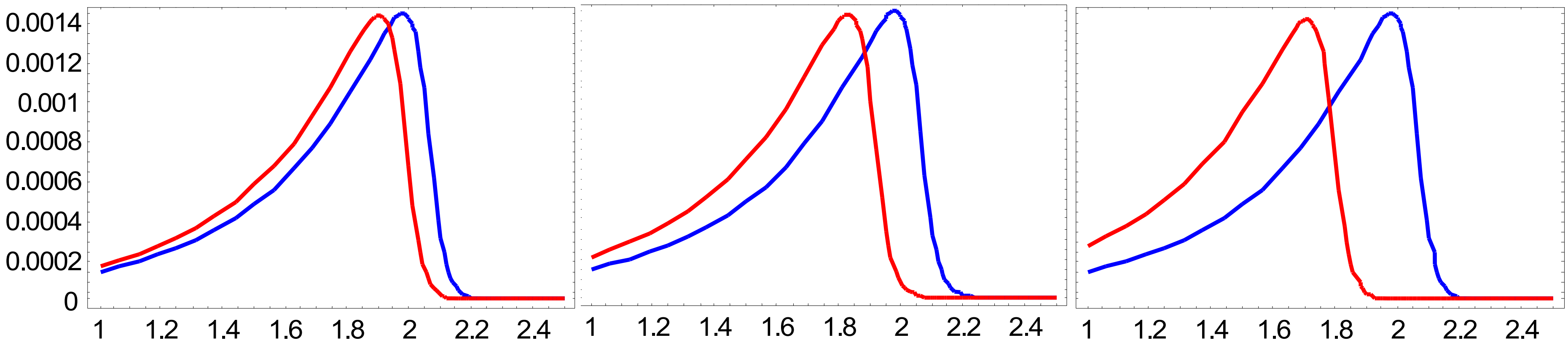
nonlinear model:

$$p(x,0,t) = \alpha(\omega_r(x) - i\partial_t)Y + B|Y|^2 Y$$



But here the extension to the general signal is complicated at least.

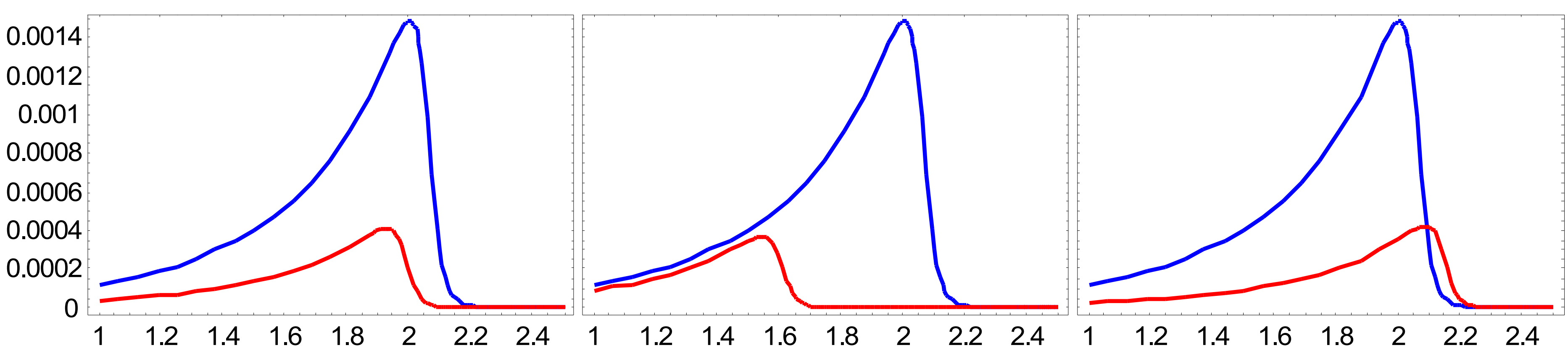
Relation to psychophysics:



1000 and 1120 Hz | 1000 and 1250 Hz | 1000 and 1500 Hz

beats and roughness | two separate tones

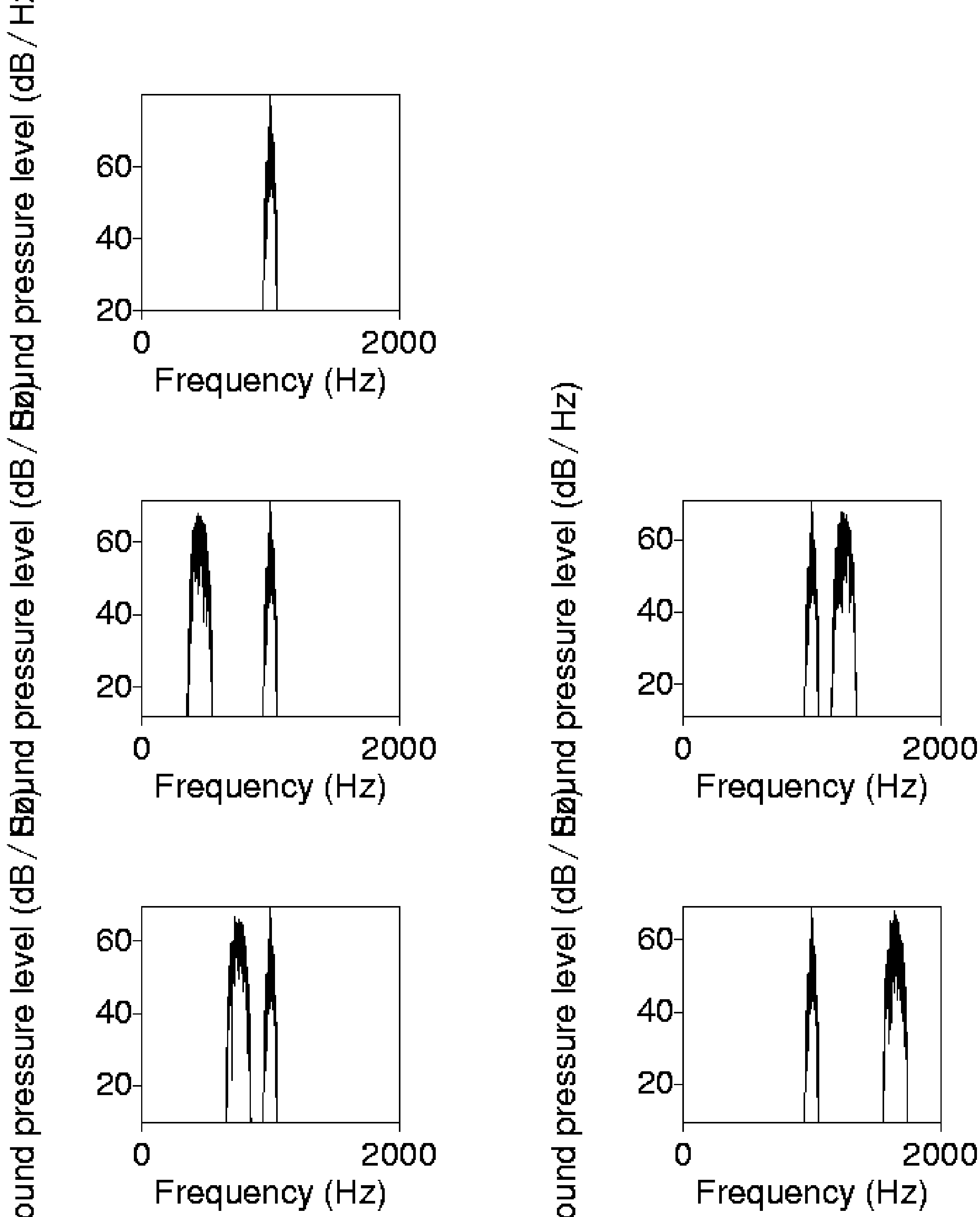
Masking:



tone of 1000 Hz masks softer tone of 1120 Hz

tone of 1000 Hz does not mask softer tone of 2000 Hz

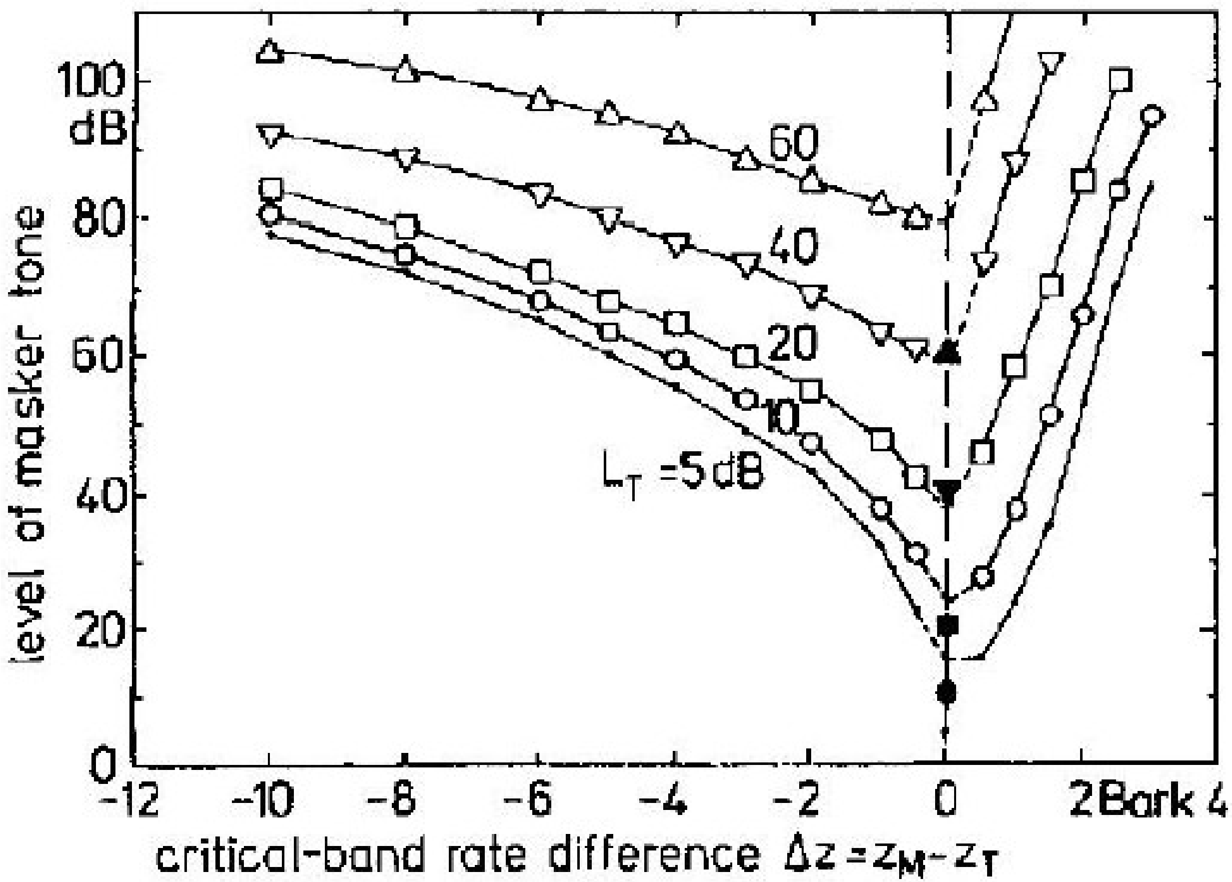
tone of 1000 Hz does not mask softer tone of 980 Hz

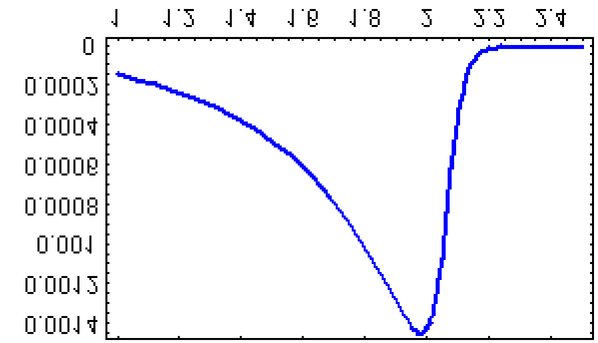


Testton

mask1 mask3
mask2 mask4

201praat: mask0 - mask4





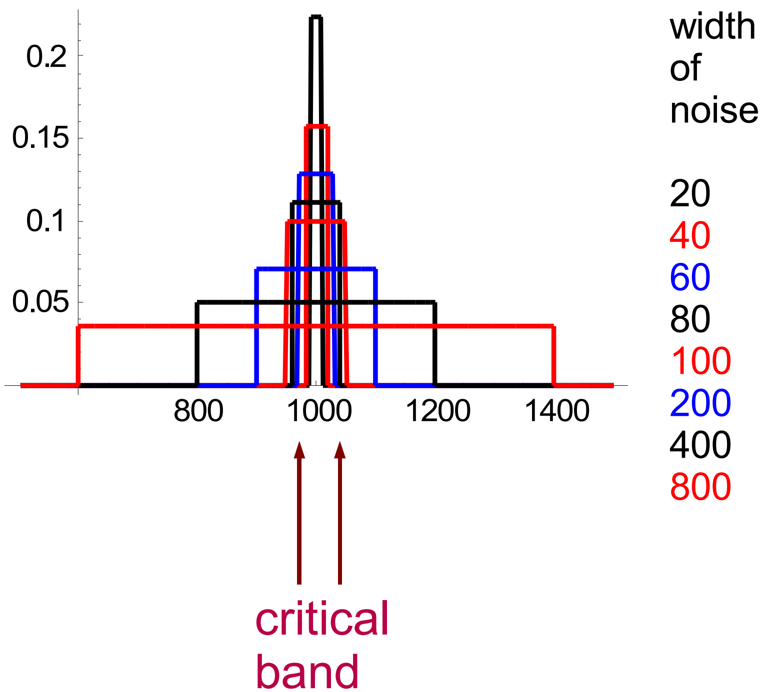
looks really like reversed excitation of bm.

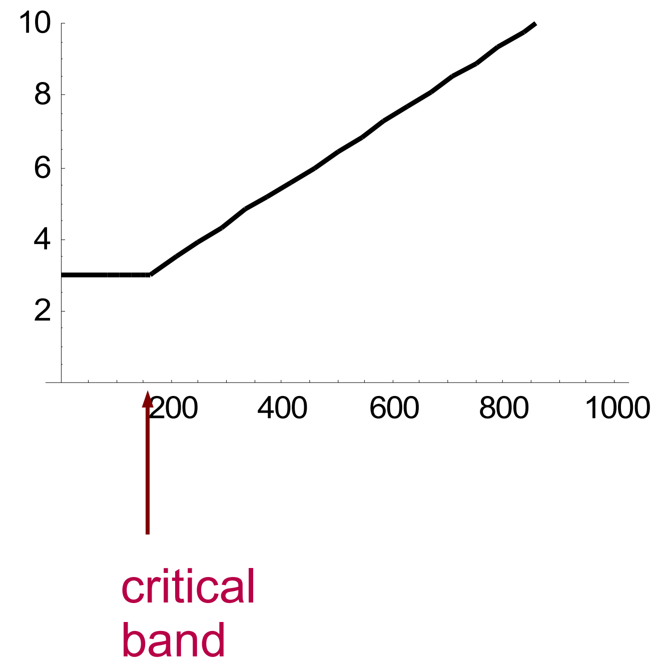
Psychoacoustic tuning curve. 1 bark corresponds about a frequency ratio of 1.16 (between a tone and 3 1/2 tones). The filled symbols indicate the volume of the test tone, the value at the corresponding open symbols the volume of the marker at threshold of extinction.

[ZF99] E. Zwicker and H. Fastl. *Psychoacoustics.* Springer, Berlin Heidelberg New York, 1999.

A tone of a fixed frequency has evidently influemce over a whole frequency region, which is called the **critical band** (Fletcher, Zwicker). It can be wiewed as the region of the bm which is affected by a tone of this frequency.

Explains also why loudeness is not a unique function of energy (sp-amplitude).
If bm is excited within one band, sensation increases much slower than linearly with energy. But if energy is distributed over more than one band, loudeness increases with the width of the excited region.





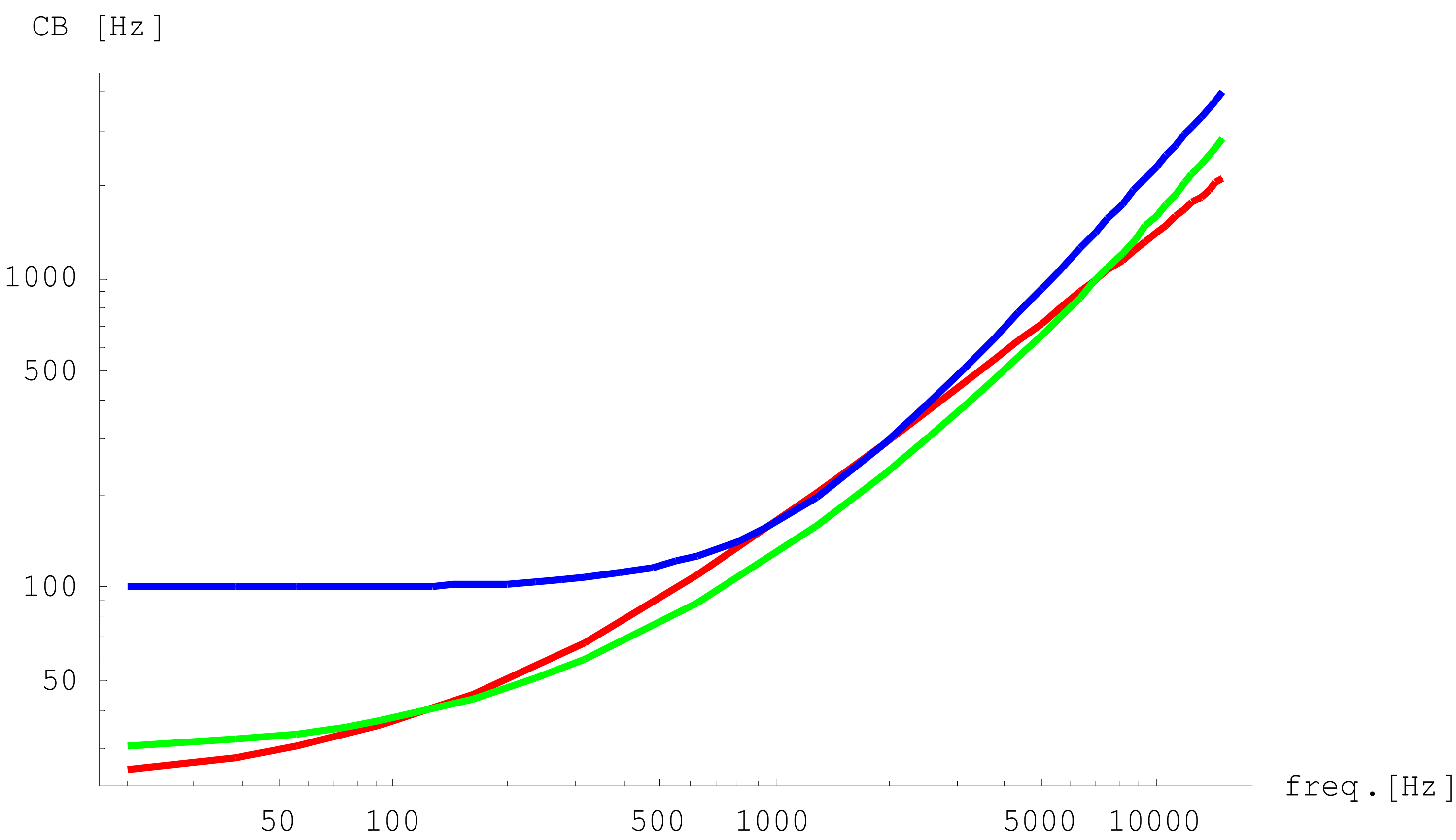
201praat noise20, noise40,... noise 800

Critical band width as function of the frequency according to different (prominent) authors:

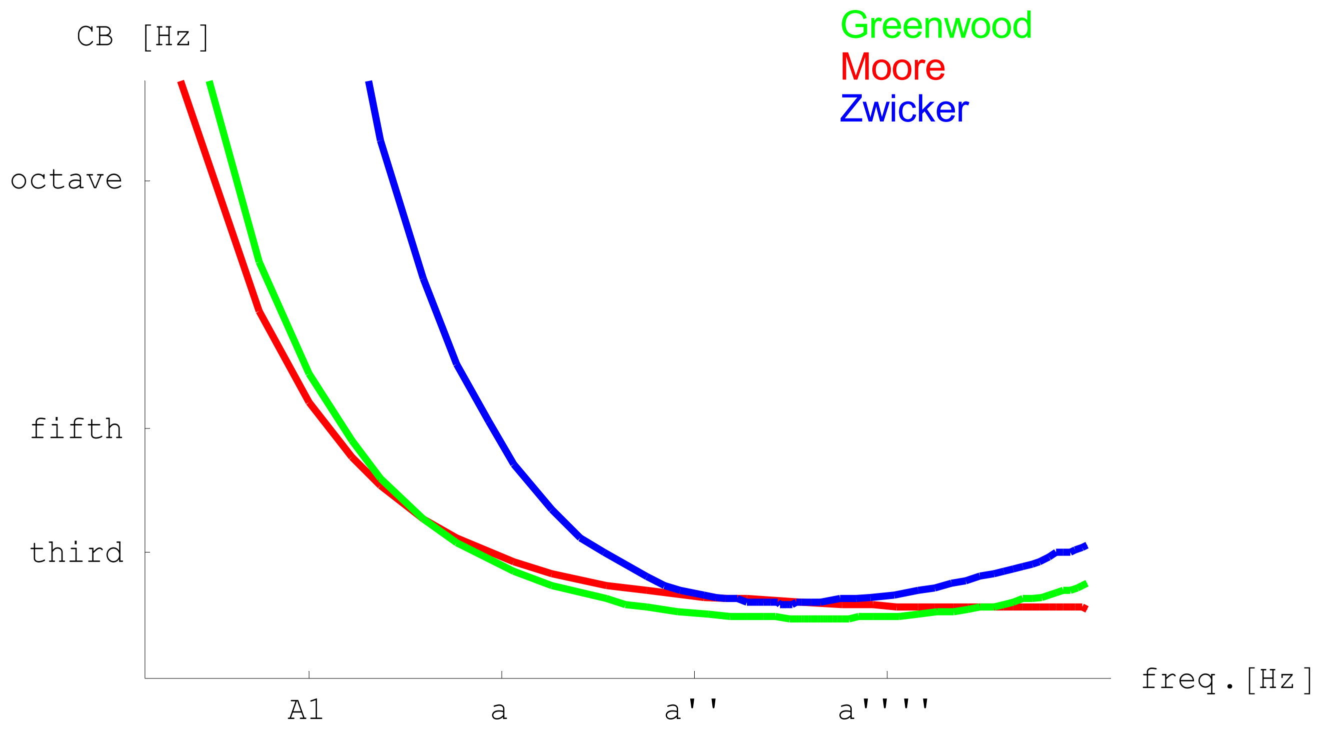
■ $22.9\,(0.006046\,x+1)$ Greenwood x: Hz

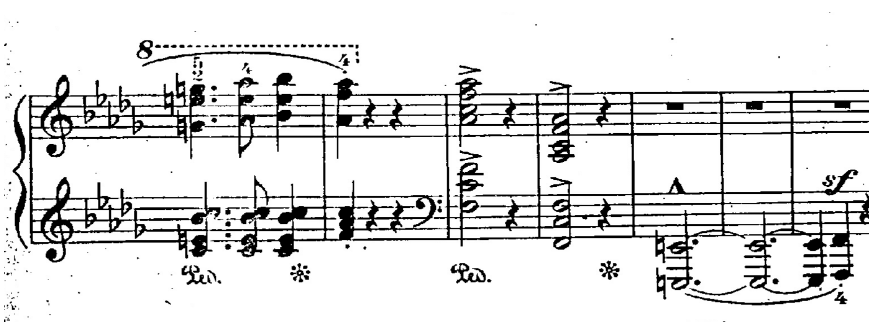
■ $0.000006231\,x^2 + 0.093391\,x + 28.52$ Moore, Glasberg

■ $75\left(1.4\left(\frac{x}{1000}\right)^2 + 1\right)^{0.69} + 25$ Zwicker, Fastl



Critical bandwidth in musical notation





Position of resonance frequency on basilar membrane

$x(\nu)$ position (from helicotrema)

$\Delta(\nu)$ critical band width

Assumption:

$$x(\nu + \Delta(\nu)) - x(\nu) = c$$

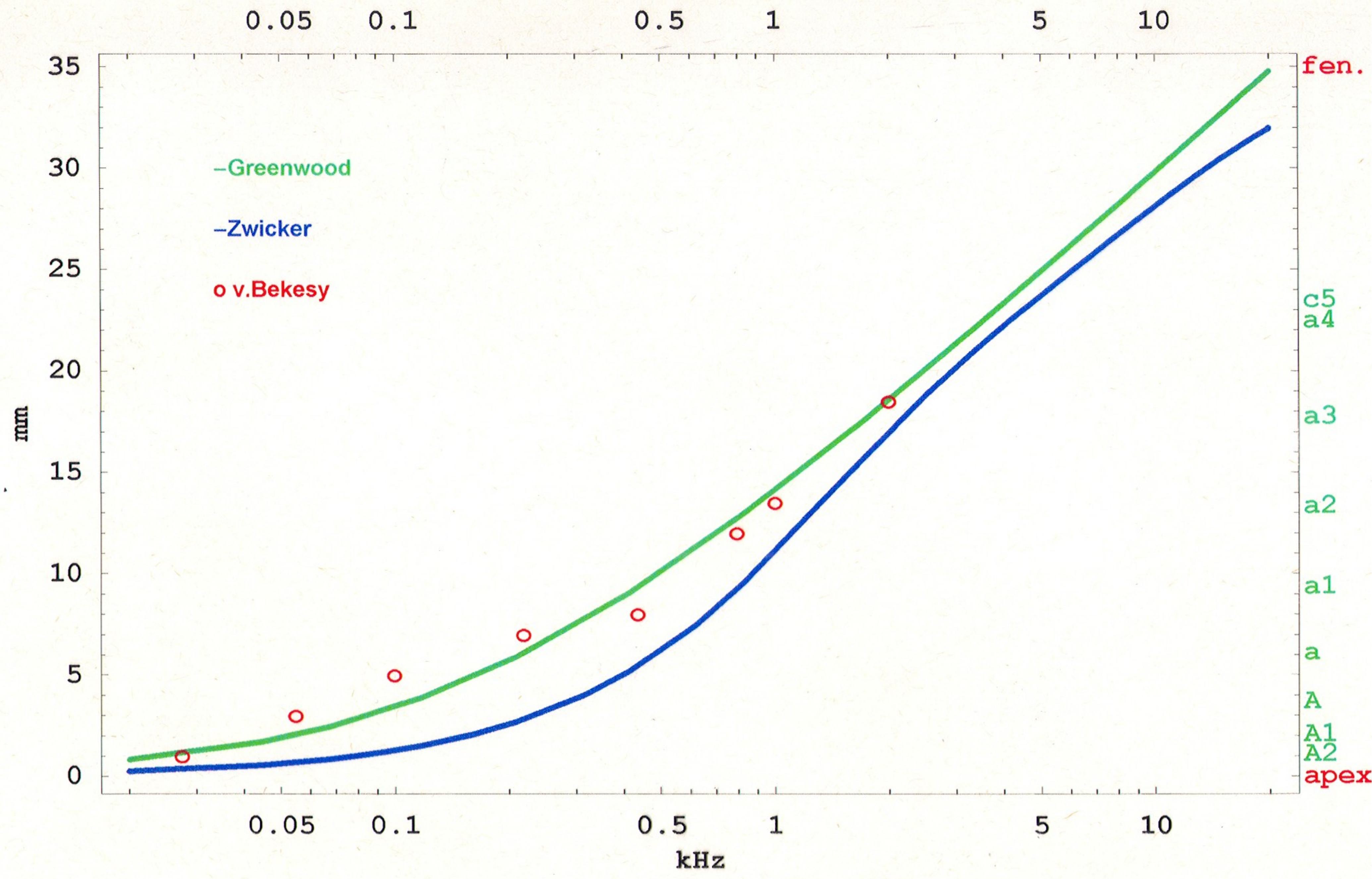
$$\frac{dx}{d\nu}\Delta(\nu) = c \quad dx = c\,\frac{d\nu}{\Delta(\nu)}$$

$$x(\nu) = c\int_{\nu_\ell}^{\nu_u} \frac{d\nu}{\Delta(\nu)}$$

$\nu_\ell = 16$ Hz; $\nu_u = 20000$ Hz

$x(\nu_u) = 3.5$ cm determines c. For Greenwood: $c = 0.1$ cm

Position of resonance frequency on the bm as obtained from critical band width, and the measurements of Bekesy



Otoacoustic Emissions

The ear is not only a receiver, but als a (very weak) transmitter of sound.

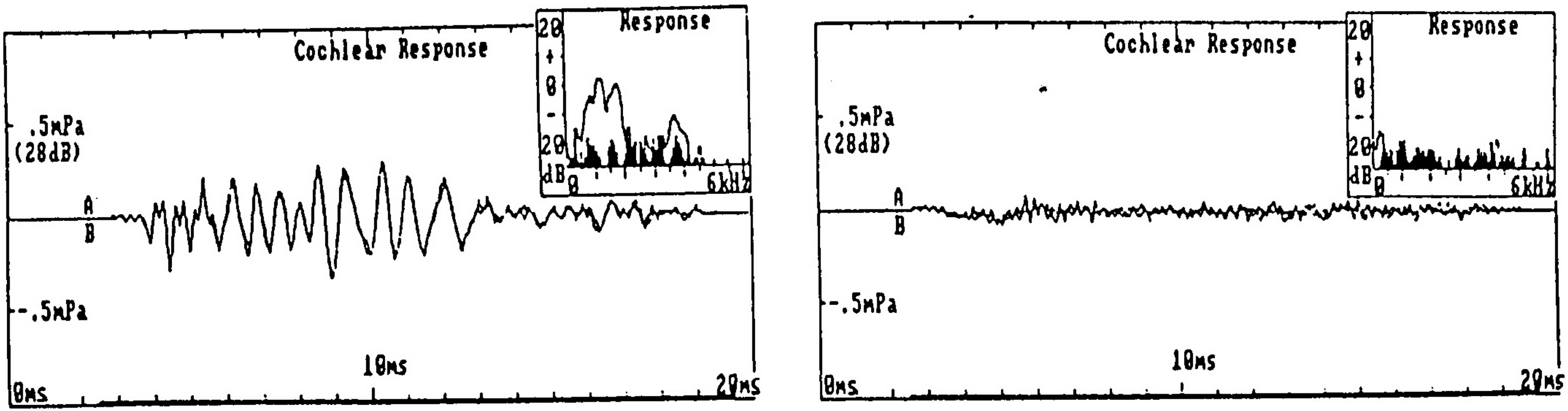


FIGURE 6.3. Transient otoacoustic emission recordings for normal ear (left) and a severely impaired ear (right) showing post stimulus ear canal waveform after averaging. Two independent measurements are superimposed to confirm reproducibility. The response power spectrum (inset) shows the cross power spectrum of the two independent responses in white, and the power spectrum of the noise, obtained by subtracting the two responses, in black. A nonlinear differential amplifier was used to eliminate stimulus artifact. (Reprinted with permission from Kemp, Ryan, and Bray 1990.)

The otoacoustic emission of the interference tone is due to the nonlinearity of the ear

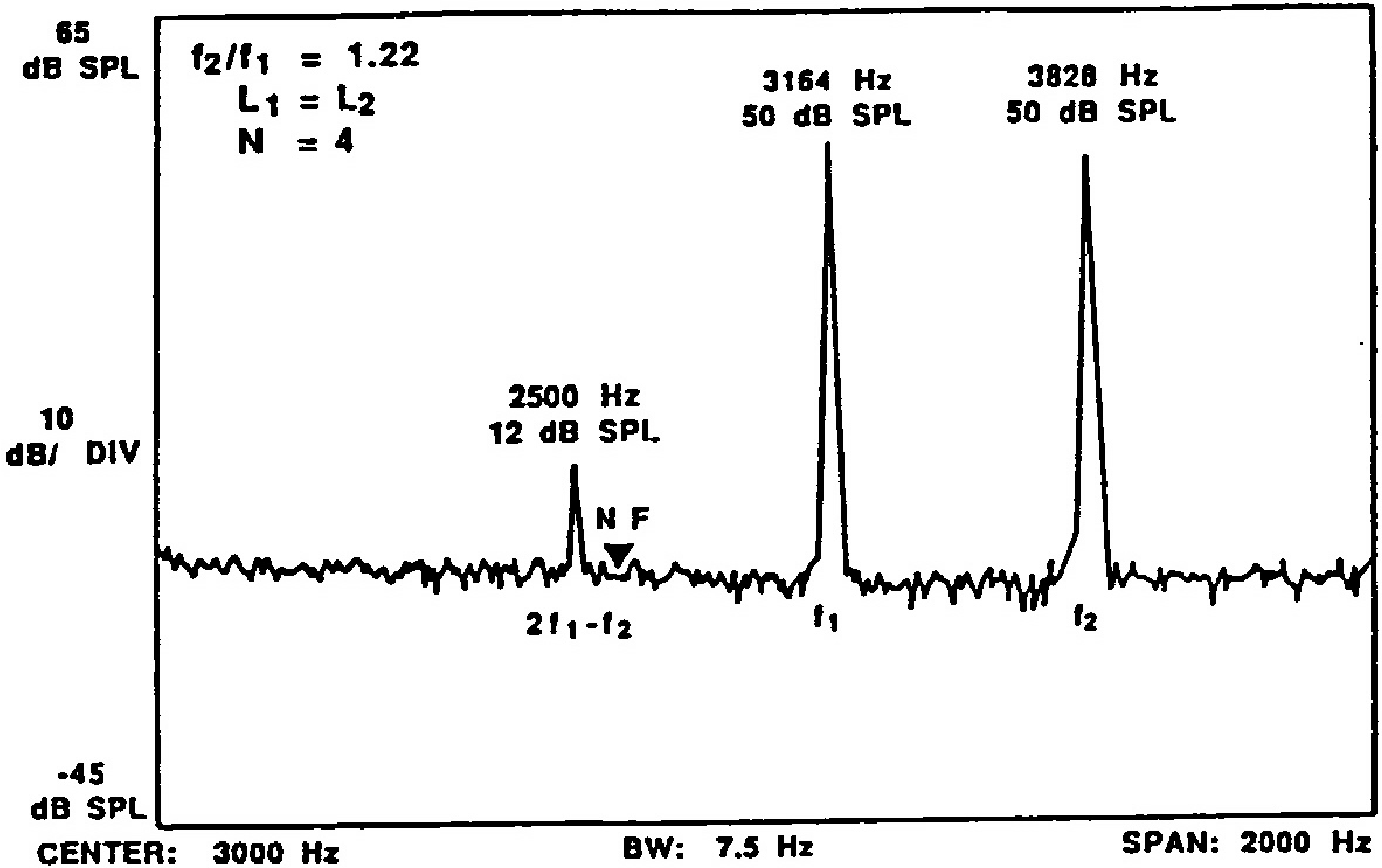


FIGURE 6.4. Example of a DPOAE spectrum. The spectral average (N = 4) of the $2f_1-f_2$ DPOAE at 2.5 kHz was in response to equilevel primaries ($L_1 = L_2$) at 50 dB SPL. Note that the amplitude of the DPOAE is about 50 dB down from the level of the primary tones. (Reprinted from Lonsbury-Martin and Martin Ear Hear 11(2) © by Williams & Wilkins 1990.)